Endogenous Structural Breaks and Real Exchange Rates Stationarity in Asia: Empirics and Theory

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Abstract

This paper reconsiders real exchange rate stationarity for six East Asian countries in the framework of the minimum Lagrangian Multiplier unit root test that accounts for multiple endogenously determined structural breaks. The result, however, is mixed. We thus set up an open-economy New Keynesian model to understand the factors that cause structural breaks and to infer the mechanism that characterizes the nonstationary real exchange rates. We find that nominal depreciation rate, inflation differentials, and interest rate differentials are the sources of breaks. The strength of each source in changing the path of real exchange rate is influenced by nominal rigidities and monetary policy reaction function. Most interestingly, we show how optimally conducted monetary policy in conjunction with endogenous currency risk premium can be the potential culprit that results in nonstationary real exchange rates.

Keywords: Real exchange rate, Purchasing power parity, LM unit root test, Structural breaks, New Keynesian model

JEL CLASSIFICATION: C22; E5; F31
I. Introduction

Purchasing power parity (PPP) as an exchange-rate determination model seems to strike back recently after going through the dark ages of voluminous disastrous empirical testing on its validity\(^1\). The central question of the debates concerning PPP reversion now has largely been directed toward the search of trend-stationary real exchange rates\(^2\). One interesting strand of literature that inspects stationarity of real exchange rates is about the use of more satisfactory econometric techniques, out of which the test of unit root null with structural breaks tops the list\(^3\).

The allowance for structural breaks in testing for unit root is not entirely new. The elegant Perron (1989) first provided a unit-root test that takes into account the presence of structural break in stationary alternative to enhance the power to reject a unit root. From single arbitrarily determined break (Perron, 1989) to endogenously determined break (Zivot and Andrew, 1992; Perron, 1997; Vogelsang and Perron, 1998), and to simultaneous two and multiple endogenous breaks (Lumsdaine and Papell, 1997; Lee and Strazicich, 2001, 2003), literature on unit root testing with structural break, especially on real exchange rate stationarity, has simply blossomed since then.

Of all the countries of interest in search for trend-stationary real exchange rates with breaks, East and Southeast Asian real exchange rates, which evidently fulfill the treatment of structural

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\(^1\) The classic Rogoff (1996), based on the numerous disappointing results of empirical validation on PPP, reached a rather gloomy assessment that PPP theory is ‘something of an embarrassment’. A more recent Taylor and Taylor (2004), however, conclude that long-run PPP may hold in the sense that there is significant mean reversion in the real exchange rates, despite the fact that short-run PPP does not hold. To see the flow of development over the years in finding PPP, one can refer to Froot and Rogoff (1995), Sarno and Taylor (2002), and Taylor (2006).

\(^2\) See Taylor (2006). For non-exhaustive list of recent works, it includes Holmes (2010) that allows for Markov switching between non-stationary and stationary regime, and between stationary regimes with different degree of persistence, Baharumshah et al. (2010) that emphasize the nonlinearity in mean-reverting process of real exchange rates, Ho et al. (2009) that couples real exchange rate stationarity to geographical factor and variability in inflation and nominal exchange rates, and Kocenda (2005) that allows for single endogenous break in real exchanges.

\(^3\) Nonlinearity in mean reversion process of real exchange rates has been another competitive candidate. One can see, for instance, Taylor and Taylor (2004), Taylor (2006), Baharumshah et al. (2010) for the pros of nonlinearity.
break especially in the aftermath of currency crash in 1997/98\textsuperscript{4}, are particularly worthwhile for inspection. The empirical results are by far encouraging, though not conclusive.

In a related study, Wu et al. (2004) found evidence of stationarity in the real exchange rates among Pacific Basin countries after considering a one-time structural change in panel data unit root tests. Zurbruegg and Allsopp (2004), by incorporating structural breaks within a multivariate cointegration structure, showed evidence in supportive of PPP. Likewise, the empirical study by Fujii (2002) suggested that the effects of the Asian crisis can be generally regarded as a temporary deviation rather than a fundamental shift in the real exchange rate behavior. Applying the Perron (1989) structural unit root test, Aggarwal et al. (2000) found clear and significant evidence in favor of PPP for most of the Southeast Asian currencies in terms of Japanese yen only when the presence of structural breaks is accommodated. Following the procedure of Johansen et al (2000) that allows for at most two predetermined breaks, Nusair (2008) provides strong support to PPP for nine Asian countries. One critical shortcoming of these studies is the arbitrarily imposition of trend breaks, or only single break point can be identified when endogenous trend break is allowed.

This paper has two goals. Firstly, at empirical front, this paper attempts to expand the literature by allowing for more than one endogenously determined structural break in the unit root testing. To this end, we use Lee and Strazicich (2001, 2003)’s minimum Lagrangian Multiplier (LM) unit root test that assumes break in null. In ADF-type unit root test that assumes no structural break under unit root null, rejection of the null can often be attributed to the presence of unit root with break, rather than the validation of stationary alternative. In contrast, Lee and Strazicich’s approach is foolproof to this pitfall, and thus allows one to draw more

\textsuperscript{4} In the tradition of Perron (1989), structural break is generally modeled either in the intercept (the crash model A) or slope (the changing-growth model B) of the trend function, or concurrently (crash-cum-growth model C).
convincing conclusion of stationarity. Out of six examined East Asian countries, the unit root null of Korea and Singapore with breaks fails to be rejected.

Secondly, at theoretical front, this paper contributes to the literature by laying out an open-economy New Keynesian model to more systematically characterize the sources of structural breaks and mechanism of unit root in real exchange rate. Most interestingly, the model attributes nonstationarity in real exchange rates to the optimally conducted monetary policy for inflation targeters like Korea in conjunction with endogenous currency risk premium.

The subsequent discussion is organized as follows. Section II reviews the idea of PPP, elaborates the inappropriateness of existing testing procedures, and illustrates our modeling strategy. We discuss the main results in Section III. It is then followed by a formalization of structural breaks in Section IV. Section V concludes.

II. Testing for Real Exchange Rate Stationarity: ADF or LM Unit-Root Test?

The purchasing power parity (PPP) hypotheses that exchange rates between two currencies adjust to reflect domestic and foreign price level (in logarithm):

\[ s_t = p_t^d - p_t^f \]  

where \( p_t^f \) and \( p_t^d \) correspondingly refers to the foreign and domestic price level, and \( s_t \) denotes the spot nominal exchange rate, defined as home over foreign currency. Early simple regression tests of absolute PPP that corresponds to Eq. (1) is given by

\[ s_t = \beta_0 + \beta_1 (p_t^d - p_t^f) + \varepsilon_t \]  

where \( \varepsilon_t \) is a disturbance term. A simple test for the absolute PPP is all about testing the null of \( \beta_1 = 1 \). On top of that, one can test a less strict form of PPP, known as relative PPP:

\[ \Delta s_t = \beta_0 + \beta_1 (\Delta p_t^d - \Delta p_t^f) + \varepsilon_t \]
where $\Delta$ denotes change in the variable. The null is $\beta_0, \beta_1 = 1$. Notwithstanding the simplicity and intuitive appeal of Eq. (3), the critical weakness is the failure to deal with the possibility of nonstationary exchange rates and prices, which, therefore, prompts an alternative approach that evaluates the stationarity of real exchange rate.

The real exchange rate $q_t$ that measures the deviation of $s_t$ from purchasing power parity can be defined as follows:

$$q_t \equiv s_t - (p^d_t - p^f_t)$$

(4)

The coefficient $\beta_1 = 1$ is now imposed not estimated. PPP holds if $q_t$ is identical to zero or a constant. Thus, the idea of PPP is closely related to the stationarity of real exchange rate. While real exchange rates may subject to considerable short run variation, the necessary condition for the PPP to hold in the long run is the stationary real exchange rate over time. Put it formally, the conditional expectation of the long run real exchange rate tends to be zero.

$$\lim_{n \to \infty} E(q_{t+n} | I_t) = 0$$

(5)

where $I_t$ is the information set available at time $t$. Accordingly, shocks to real exchange rate disappear as the time horizon expands. Therefore, the autoregressive representation of $q_t$ must have no unit root.

That said, we consider the following data generating process with a $p$-order autoregressive process to implement a test on stationarity in real exchange rate:

$$q_t = r_0 + \sum_{j=1}^{p} r_j q_{t-j} + \varepsilon_t$$

(6)

where the error term $\varepsilon_t$ follows a white noise process. As shocks are transitory, the real exchange rate will settle down to its long run equilibrium level $q^*$, where

$$q^* = \frac{r_0}{1 - \sum_{j=1}^{p} r_j}$$

(7)
If $\sum_{j=1}^{p} y_j = 1$, then the process that drives $q_t$ contains unit root, and shock to the real exchange rate is permanent. In consequence, the long run equilibrium does not exist, as division by zero is undefined in Eq. (7). To the contrary, if the unit root hypothesis is rejected and $\sum_{j=1}^{p} y_j < 1$, then shock to the real exchange rate vanishes in the long run at the rate of $1 - \sum_{j=1}^{p} y_j$ per time period.

We reparameterize Eq. (6) to produce the following empirical equation

$$\Delta q_t = \gamma_0 + \rho q_{t-1} + \sum_{j=1}^{p} \gamma_j q_{t-j} + \epsilon_t$$

(8)

The most commonly used test is the augmented Dickey-Fuller (ADF) test proposed by Dickey and Fuller (1981) under the unit root null hypothesis of $\rho = 0$. Rejection of the null suggests little tendency for deviations from PPP.

II.1 Why Not ADF-Type Unit Root Test?

One limitation of conventional ADF unit root test is the inability to recognize PPP when it actually holds outside the structural shift. Aggarwal et al. (2000), Narayan (2005), and Papell and Prodan (2006), among many others, have convincingly shown that the introduction of structural break(s) makes the existence of PPP across countries more clear.

Although the recent ADF-type unit root test is remarkably able to accommodate for multiple endogenous trend break (see, for instance, Lumsdaine and Papell, 1997), these tests assume no structural break under the unit root null, and derive the critical values accordingly. Nunes et al. (1997) showed that this assumption can result in spurious rejections should there is a break under the null.

Lee and Strazicich (2001) further argued that ADF-type endogenous break test tends to incorrectly select the break point, and more so should the magnitude of the break increases. At this misspecified break point, bias in estimating the persistence is maximized, resulting in
spurious rejection of the unit root null. Accordingly, Lee and Strazicich (2003) proposed a one-break LM unit root test as an alternative to Zivot-Andrews test, and a two-break LM unit root test for Lumsdaine-Papell test. Contrary to the ADF-type test, the size properties of LM unit root test are unaffected by the presence of breaks under the null. As such, LM unit root test, which allows for structural break under both unit root null and stationary alternative, is able to obviate invalid estimation of break point and spurious rejection. For this reason, this paper adopts Lee and Strazicich (2003) LM unit root test with two breaks.

II.2 LM Unit Root with Two Endogenous Structural Breaks

Consider Lee and Strazicich (2003) procedure that corresponds to Perron (1989) Model C that allows for two changes in both level, $D_{jt}$, and trend, $DT_{jt}$, in the series for $j = 1, 2$. The two-break LM unit root test statistic for real exchange rates stationarity can be obtained from the following regression:

$$\Delta q_t = \phi' \Delta Z_t + \phi \tilde{S}_{t-1} + e_t$$  \hspace{1cm} (9)

where the vector of exogenous variables, $Z_t$, takes the form $[1, t, D_{1t}, D_{2t}, DT_{1t}, DT_{2t}]'$. For $t \geq T_{Bj} + 1$, the dummy for level shift equals one $D_{jt} = 1$, whereas the dummy for trend shift $DT_{jt}$ equals $t - T_{Bj}$, or zero otherwise. The parameter $T_{Bj}$ refers to the time location of the break. $\tilde{S}_t$ is detrended series such that $\tilde{S}_t = q_t - q_1 - (Z_t - Z_1)\tilde{\delta}$ for $t = 2, \ldots, T$. $\tilde{\delta}$ denotes coefficients in the regression of $\Delta q_t$ on $\Delta Z_t$, while $q_1$ and $Z_t$, respectively, represents the first observations of $q_t$ and $Z_t$. To account for autocorrelation problem the regression is augmented with the lagged dependent variables.

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5 We do not consider model B as it is commonly believed that most economic time series can be adequately accommodated by model A and C. Sen (2003) suggested that model C yields more reliable estimates as compared to model A, and the former is thus preferred to the latter when break point is endogenously determined (see, also, Narayan, 2005). For this reason, we consider only model C.
The unit root null hypothesis $\phi = 0$ is tested by the $t$-statistics $\tau$. The unknown break dates $T_{Bj}$ for $j = 1, 2$ are determined endogenously by searching over the trimming region of $(0.1T, 0.9T)$, where $T$ is the sample size. By denoting the break fractions $\lambda_j$ as $T_{Bj}/T$ when $T \to \infty$, the LM test statistics is defined as:

$$LM_\tau = \inf_{\lambda} \tilde{t}(\lambda)$$

(10)

The critical values for the two-break minimum LM unit root test statistics are tabulated in Lee and Strazicich (2003).

III. Results and discussion

In this section, we examine real exchange rates of six East Asian economies, namely Indonesia, Korea, Malaysia, Philippines, Singapore, and Thailand over the time period January 1993 to July 2010. Here we treat U.S dollar as the numeraire since U.S has long been Asian major trading partner, particularly as the final destination for regional produces, not to speak its role as the dominant vehicle currency in international trade (Goldberg and Tille, 2008). To construct real exchange rates as defined in Eq. (4), monthly data on average market nominal exchange rates and consumer price index are extracted from International Financial Statistics (IFS) data bank.

Figure 1 illustrates the variability of real exchange rates for the six Asian economies throughout the years of 1993 and 2010. Sudden and drastic upsurge in real exchange rates during Asian crisis apparently dominates the scene, suggesting a potential structural shift in the mean and trend of the data after 1997. It is thus interesting to know whether Asian real exchange rates are characterized by unit root or indeed trend-stationary that is just subject to structural break.

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6 Nusair (2008) also empirically concluded that base currency is not critical to the finding of long-run PPP for Asian countries.
Equally fascinating is the fact that to find another break through informal eyeballing of Figure 1 without hesitation is barely possible. This difficulty suggests that allowing the breaks to be endogenously determined is a sensible implementation.

[Figure 1] Real Exchange Rates (in natural logarithms), 1993:M01 – 2010:M07
We begin the empirical analysis with the ADF test that serves as a benchmark for comparison. The test includes a deterministic trend to accommodate the potential time trends in real exchange rates driven by the relative consumer prices should Balassa-Samuelson effect presents. Using the ‘general-to-specific’ recursive $t$-statistics procedure, we set the maximum value of $k$ equal to 12 and use approximate 10% critical value of 1.645 from the asymptotic normal distribution to assess significance of the last lag. Ng and Perron (1995) demonstrated that such a ‘$t$-sig’ approach produces test statistics that performs better in terms of size and power than information-based criteria such as the Akaike information Criterion or the Schwartz Bayesian Criterion. The results from the ADF test, with and without a trend, are reported in Table 1.

**Table 1** ADF test results

<table>
<thead>
<tr>
<th>Country</th>
<th>$k$</th>
<th>$\rho$</th>
<th>$\delta$</th>
<th>$k$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indonesia</td>
<td>11</td>
<td>-1.6941</td>
<td>-0.6605</td>
<td>5</td>
<td>-2.5072</td>
</tr>
<tr>
<td>Korea</td>
<td>9</td>
<td>-2.3236</td>
<td>0.3942</td>
<td>9</td>
<td>-2.3138</td>
</tr>
<tr>
<td>Malaysia</td>
<td>8</td>
<td>-1.2004</td>
<td>-0.1793</td>
<td>8</td>
<td>-1.7786</td>
</tr>
<tr>
<td>Philippines</td>
<td>8</td>
<td>-1.2998</td>
<td>0.0351</td>
<td>8</td>
<td>-1.4580</td>
</tr>
<tr>
<td>Singapore</td>
<td>5</td>
<td>-0.3280</td>
<td>-0.8293</td>
<td>5</td>
<td>-1.0208</td>
</tr>
<tr>
<td>Thailand</td>
<td>10</td>
<td>-1.0118</td>
<td>-0.6462</td>
<td>10</td>
<td>-1.5023</td>
</tr>
</tbody>
</table>

**Note:** $\rho$ is the coefficient on the one-period lagged level of the real exchange rate variable. $\delta$ is the coefficient on the time trend. $k$ is the optimal lag length selected using general to specific approach.

As expected, the unit root null cannot be rejected. One typical response to such findings is to fault the tests for their lack of power. But as noted previously, the failure to incorporate structural changes in testing the unit root of a time series is too often biased toward finding nonstationarity (see Rappoport and Reichlin, 1989). In view of the potential instability of the PPP relationship associated with exceptional events like the onset of Asian currency crisis, unit root tests that permit level and trend break are naturally sensible for analyzing the long run behavior of the real
exchange rates. Indeed, by applying the LM unit root test with two endogenous breaks, we find some evidence in favor of the PPP hypothesis. The results of the two-break LM unit root test are presented in Table 2.

<table>
<thead>
<tr>
<th>Country</th>
<th>( k )</th>
<th>( TB_{1t} )</th>
<th>( TB_{2t} )</th>
<th>Test statistics</th>
<th>( \Delta D_{1t} )</th>
<th>( \Delta D_{2t} )</th>
<th>( \Delta DT_{1t} )</th>
<th>( \Delta DT_{2t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indonesia</td>
<td>08</td>
<td>1997M10</td>
<td>1999M03</td>
<td>-0.3967</td>
<td>-0.2454</td>
<td>0.1088</td>
<td>0.2027</td>
<td>-0.1087</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-8.0040)***</td>
<td>(-3.5716)***</td>
<td>(1.5686)</td>
<td>(6.5272)***</td>
<td>(-5.1889)***</td>
</tr>
<tr>
<td>Korea</td>
<td>09</td>
<td>1997M09</td>
<td>2005M01</td>
<td>-0.2410</td>
<td>0.0082</td>
<td>-0.0069</td>
<td>0.0610</td>
<td>-0.0442</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-4.9333)</td>
<td>(0.1851)</td>
<td>(-0.1594)</td>
<td>(4.1278)***</td>
<td>(-3.7591)***</td>
</tr>
<tr>
<td>Malaysia</td>
<td>07</td>
<td>1997M06</td>
<td>1999M06</td>
<td>-0.2400</td>
<td>-0.0308</td>
<td>0.0014</td>
<td>0.0514</td>
<td>-0.0325</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-5.6562)**</td>
<td>(-1.3619)</td>
<td>(0.0668)</td>
<td>(6.1405)***</td>
<td>(-5.7425)***</td>
</tr>
<tr>
<td>Philippines</td>
<td>09</td>
<td>1997M07</td>
<td>2004M02</td>
<td>-0.1942</td>
<td>0.0091</td>
<td>0.0264</td>
<td>0.0296</td>
<td>-0.0361</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-5.4793)</td>
<td>(0.4376)</td>
<td>(1.2881)</td>
<td>(5.5873)***</td>
<td>(-6.5580)***</td>
</tr>
<tr>
<td>Singapore</td>
<td>11</td>
<td>1997M08</td>
<td>2003M06</td>
<td>-0.1668</td>
<td>-0.0075</td>
<td>0.0017</td>
<td>0.0255</td>
<td>-0.0116</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-4.2282)</td>
<td>(-0.4903)</td>
<td>(0.1130)</td>
<td>(4.4373)***</td>
<td>(-4.3730)***</td>
</tr>
<tr>
<td>Thailand</td>
<td>09</td>
<td>1997M07</td>
<td>2003M01</td>
<td>-0.3355</td>
<td>-0.0618</td>
<td>0.0237</td>
<td>0.0599</td>
<td>-0.0325</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-6.1166)**</td>
<td>(-2.0925)***</td>
<td>(0.9064)</td>
<td>(5.6123)***</td>
<td>(-5.6838)***</td>
</tr>
</tbody>
</table>

Table 2 reveals that the unit root null can be rejected in favor of the trend-break stationary alternative for four of the six real exchange rates (Korea and Singapore are the exceptions), out of which three is at least at 5% significant level. As the minimum LM tests assume two breaks under the null, rejection of unit root null profoundly confirms that real exchange rates of these Asian countries are trend-stationary that are subject to structural change. Interestingly, the results also indicate that trend break is statistically more critical than level-break in accounting for real exchange rate stationary.

Because minimum LM test endogenizes the determination of break points, the outcome is foolproof to the criticism of arbitrarily imposed break dates (Christiano, 1992). The first estimated break points collectively lie in the June and October of 1997. This finding is certainly consistent with the onset of Asian currency and financial crises since July 1997. More puzzling is the heterogeneous second estimated break points that fall unsystematically in between 2003 and 2005. Unlike the first estimated break points, these second estimated breaks are not cursorily
observable in the figure, which of course substantiates the importance of endogeneity in break
determination. Of further interest is whether mechanism that is able to consistently account for
this variety of break points can be found.

IV. An interpretation of structural breaks in New Keynesian model

Conceptually, the path of real exchange rate (in logarithm) can be described as

\[ q_t = (1 - \rho) q^* + \rho q_{t-1} + \epsilon_t \]  

(11)

where \( q^* \) denotes the steady-state (long-run average) real exchange rate, and \( \epsilon_t \sim iid(0, \sigma^2) \) is the
stochastic white noise. Let \( \hat{q}_t = q_t - q^* \). We can augment Eq. (11) to conceptually resemble
Perron (1989)’s model C of structural break (ignoring time trend and constant):

\[ \hat{q}_t = \rho \hat{q}_{t-1} + \mu_1 D_P + \mu_2 D_L + \sum_{i=1}^{k} \phi_{2+i} \Delta q_{t+i} + \epsilon_t \]  

(12)

where \( D_P \) and \( D_L \), respectively, denotes pulse dummy and level dummy. In line with Eq. (8) in
section II, Eq. (12) indicates that a temporary innovation can end with permanent deviation of
real exchange rate from steady state if \( \rho \geq 1 \). It is thus of interest to figure out the structural
meaning of \( \rho \). Besides, what constitute pulse dummy and level dummy? Can we structurally
define \( \mu_1 \) and \( \mu_2 \)? To this end, we set up a simple open-economy New Keynesian model model
to provide a body of analytics for inspecting the mechanism of breaks in real exchange rates.

IV.1 A New Keynesian set-up

Consider an economy with working households endowed with one-period state-contingent
bonds. Human and non-human incomes finance non-storable consumption \( (G_t) \) and purchase of
assets. Household’s problem, therefore, can be written as to maximize the discounted future
streams of utility,
\[ u = E \left[ \sum_{t=0}^{\infty} \beta^t \left( \frac{c_{t+1}}{1-\sigma} - \log N_t \right) \right] \]  

subject to the budget constraint

\[ b_t + Q_t b_t^f = (1 + r_{t-1}) b_{t-1} + Q_t (1 + r_t^f) \Phi(s_{t-1}) b_{t-1}^f + W_t N_t - C_t \]  

where \( \beta \) is the discounted rate. \( Q_t (\equiv S_t P_t^f / P_t) \) is the real exchange rate, defined as nominal exchange rate adjusted by the relative consumer prices. \( \Phi(s_{t-1}) \) is risk premium that is increasing in the level of nominal exchange rate, \( \Phi'(s_{t-1}) > 0 \). It indicates that current level of nominal exchange rate will influence the prospective risk premium of the currency, which, in turn, affect the path of future nominal exchange rates.

On top of that, \( b_t \) and \( b_t^f \), respectively, denotes real domestic and foreign bonds that pay real yield. \( W_t \) is real wage and \( N_t \) is hours worked. The parameter \( \sigma \) is the coefficient of relative risk aversion. Its reciprocal thus denotes the elasticity of substitution between consumptions at \( t \) and \( t + 1 \). For the sake of simplicity without implication, we assume unitary wage elasticity of labor supply. Consumption goods is a composite of home and imported foreign goods

\[
C_t = \left[ (\gamma)^{1/\omega} (C_{H,t})^{(\omega-1)/\omega} + (1 - \gamma)^{1/\omega} (C_{F,t})^{(\omega-1)/\omega} \right]^{\omega/(\omega-1)}
\]  

where \( C_{H,t} = \left[ \int_0^1 \{C_{H,t}(j)\}^{(e-1)/e} dj \right]^{e/(e-1)} \) and \( C_{F,t} = \left[ \int_0^1 \{C_{F,t}(j)\}^{(e-1)/e} dj \right]^{e/(e-1)} \).

Derived from the first order conditions, the marginal rate of substitution between consumption and hours worked is given by

\[
(C_t)^\sigma = W_t N_t
\]  

Besides, consumer's optimality condition with respect to intertemporal consumption allocation can be derived as

\[
\frac{1}{1+i_t} = \beta E \left[ \left( \frac{c_{t+1}}{c_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right]
\]
where $i_t$ is nominal interest rate, and $P_t$ is price level. Under the assumption of complete securities market, Eq. (17) holds equally for foreign representative consumer. Taken together with arbitrage in securities that assures international risk sharing

$$\frac{1+i_t^f}{1+i_t} = \left(\frac{Q_t}{Q_{t+1}}\right) \left(\frac{P_{t+1}^f}{P_t^f}\right) \left(\frac{P_t}{P_{t+1}}\right) \Phi(S_{t-1})^{-1}$$

and the definition of real exchange rate, we can get

$$C_t = C_t^f \left(\frac{Q_t}{Q_t^f}\right)^{\frac{1}{\sigma}}$$

Consider now a simple production function of firm $j$ with time-to-hire labor.

$$Y_t(j) = A_t N_{t-1}(j)$$

By denoting the shadow price of Lagrangian problem as real marginal cost ($RMC_t$), firm’s cost minimization problem gives the following optimal condition.

$$W_{t-1} N_{t-1}(j) = RMC_t Y_t(j)$$

Firm $j$ has to choose $P_t^j$ in order to maximize the expected discounted profits ($E_t \Pi_t$) in the form

$$E_t \Pi_t = E_t \left\{ \sum_{i=0}^{\infty} (\theta t)^i \Lambda_{t+i} \left[ \frac{p_t^j(j)-MC_{N,t}}{P_{t+i}} \right] \left[ \frac{p_t^j(j)}{P_t} \right]^{-\sigma} \left[ C_{H,t} + C_{H,t}^f \right] \right\}$$

where $\theta_p$ is the probability that firm $j$ is not able to adjust price. $\Lambda_{t+i}$ satisfies $(C_{t+1+i}/C_{t+i})^{-\sigma}$, $\beta^{i} C_{t+1+i}^{-\sigma}$ denotes the discounted factor in the interest of households. $MC_{N,t}$ is consumer-good producer’s nominal marginal cost, and $C_{H,t}^f$ is home export. Eq. (22) assumes producer currency pricing in export, and the pricing decision is symmetry for all firm $j$ in equilibrium.

In Calvo price setting, firms that receive signals at probability $1 - \theta$ for price reoptimization will reset price to approximate optimal reset price ($P_t^j$) derived from Eq. (22). Those that have not received the signal at probability $\theta$ will follow last-period price, out of which a fraction ($\Psi$)
will index their new price to last-period inflation. Since aggregate log price level at each date is a probability-weighted average of partially indexed aggregate log price level and newly reset price, the New Keynesian Phillips curve for inflation can be derived as

\[
\pi_t = \left\{ \frac{\psi}{1+\theta \beta \psi} \right\} \pi_{N,H,t-1} + \left\{ \frac{\beta}{1+\theta \beta \psi} \right\} E_t \pi_{t+1} + \left\{ \frac{(1-\theta)(1-\theta \beta)}{\theta(1+\theta \beta \psi)} \right\}(r\bar{m}_t)
\]  

(23)

To close the model, we assume the following monetary policy function:

\[
i_t = \beta^{-1} - 1 + (1 - v_\pi) \pi^* + v_\pi \pi_t + v_y \hat{y}_t + v_q \hat{q}_t + v_{\Delta q} \Delta q_t
\]  

(24)

The sum of first two items on the right hand side amounts to targeted nominal interest rate. The coefficient \(v_\pi\), \(v_y\), \(v_q\), and \(v_{\Delta q}\), respectively, indicates policy reaction towards stabilization of inflation, output, and level and changes in real exchange rates. Note that all corresponding behavioral equations derived from consumption Euler condition to inflation dynamics and monetary policy is equally applicable for foreign country.

IV. 2 Deriving the sources of unit root and structural breaks

Consider the definition of real exchange rate. The log-deviation of real exchange rates can be written as

\[
\hat{q}_t = \hat{q}_{t-1} + \Delta S_t + \pi^f_t - \pi_t
\]  

(25)

The inflation differentials in Eq. (25) can be substituted by the difference between New Keynesian Philips curve of foreign and home economy, which simplifies to

\[
\hat{q}_t = \hat{q}_{t-1} + \Delta S_t + \varphi_1(\pi^f_{t-1} - \pi_{t-1}) + \varphi_3(\pi^f_{t+1} - \pi_{t+1}) + \varphi_2(r\bar{m}_c^f - r\bar{m}_c)
\]  

(26)

where \(\varphi_1 = \frac{\psi}{1+\theta \beta \psi}\), \(\varphi_2 = \frac{(1-\theta)(1-\theta \beta)}{\theta(1+\theta \beta \psi)}\), and \(\varphi_3 = \frac{\beta}{1+\theta \beta \psi}\).

Next, Pareto efficient allocation ensures that

\[
\sigma \hat{c}_{t-1} = r\bar{m}_c + \hat{y}_t
\]  

(27)
Together with the foreign counterpart of Eq. (27) and Eq. (19), Eq. (26) can be further simplified to

$$\hat{q}_t = (1 - \varphi_2)\hat{q}_{t-1} + \Delta S_t + \varphi_1(\pi^f_{t-1} - \pi_{t-1}) + \varphi_3(\pi^f_{t+1} - \pi_{t+1}) + \varphi_2(\hat{y}_t - \hat{y}^f_t) \quad (28)$$

The last resolution is to put monetary policy into Eq. (28). Rearrange home monetary policy function of Eq. (24) and identical foreign monetary policy function in such a way that

$$\hat{y}_t - \hat{y}_t^f = \frac{1}{v_y} [i_t - i_t^f + v_\pi (\pi^f_t - \pi_t) - (v_q + v_q^f) \hat{q}_t - (v_{\Delta q} + v_{\Delta q}^f) \Delta q_t] \quad (29)$$

Assume that $v_q^f = v_{\Delta q}^f = 0$ since neither the level nor the rate of changes in real exchange rate has never been the concern for U.S. monetary policymaker. Substitute Eq. (29) into Eq. (28), we get

$$\hat{q}_t = \left\{ \frac{(1 - \varphi_2)v_y}{v_y + \varphi_2v_q} \right\} \hat{q}_{t-1} + \left\{ \frac{\varphi_2}{v_y + \varphi_2v_q} \right\} (i_t - i_t^f) + \left\{ \frac{v_y}{v_y + \varphi_2v_q} \right\} \Delta S_t$$

$$+ \left\{ \frac{v_y \varphi_1}{v_y + \varphi_2v_q} \right\} (\pi^f_{t-1} - \pi_{t-1}) + \left\{ \frac{v_y \varphi_3}{v_y + \varphi_2v_q} \right\} (\pi^f_{t+1} - \pi_{t+1})$$

$$+ \left\{ \frac{\varphi_2 v_\pi}{v_y + \varphi_2v_q} \right\} (\pi^f_t - \pi_t) - \left\{ \frac{\varphi_2 v_{\Delta q}}{v_y + \varphi_2v_q} \right\} \Delta q_t \quad (30)$$

It can be easily shown that Eq. (30) actually resembles Eq. (12)

$$\hat{q}_t = \left\{ \frac{(1 - \varphi_2)v_y}{v_y + \varphi_2v_q} \right\} \hat{q}_{t-1} + \left\{ \frac{\varphi_2}{v_y + \varphi_2v_q} \right\} (i_t - i_t^f)$$

$$+ \left\{ \frac{1}{v_y + \varphi_2(v_{\Delta q} + \Delta q)} \right\} \left\{ (v_y - \varphi_2 v_\pi) \Delta S_t - v_y \varphi_1 \Delta S_{t-1} - v_y \varphi_3 \Delta S_{t+1} \right\}$$

$$+ \left\{ \frac{1}{v_y + 2\varphi_2(v_q + \Delta q)} \right\} \sum_{l=-1}^{1} \varphi_{2+l} v^{|l|}_y (v_\pi - v_{\Delta q})^{1-|l|} \Delta q_{t+l} \quad (31)$$

in that the coefficients $\rho$, $\mu_1$, and $\mu_2$ of Eq. (12) now have their structural meanings.

IV.3 What causes breaks?
According to Eq. (30), interest rate differentials due to asymmetry in the conduct of monetary policy constitute the break in the level of real exchange rate. Nominal rate of depreciation and inflation differentials contribute to the break in steepness of the trend of real exchange rate. The magnitude of each source of break depends on the degree of price stickiness, which influences the value of $\phi_t$, and policy reaction toward targeted variables. Although it is beyond our scope to provide precise quantitative estimates of the strength of each source, some simple simulations on Eq. (30) allow us to infer how change in nominal rigidities and the conduct of monetary policy would have affected the potential occurrence of breaks.

In particular, results contained in Table 3 show that

(i) Under a flexible-price environment, nominal depreciation rate, lagged and expected inflation differentials are barely relevant in the path of real exchange rate. To the contrary, interest rate differentials and contemporaneous inflation differentials become more important as a source of level- and mean-break.

(ii) Should the policymaker places greater weight on the stabilization of real exchange rate variability, the magnitudes of all sources of break are apparently moderated, which, in turn, implies a declining likelihood of structural change to occur.

(iii) More hawkish attitude toward inflation stabilization enhances the magnitude of contemporaneous inflation differentials as source of mean-break.

(iv) Stronger output stabilization drastically reduces the magnitude and thus the role of differentials in interest rates and contemporaneous inflations, while moderately enhancing the nontrivial effect of nominal depreciation rate, and lagged and expected inflation differentials in the trend of real exchange rate.
[Table 3] The effects of nominal rigidities and monetary policy in generating structural breaks

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Baseline</th>
<th>Flexible price</th>
<th>Targeting Δ$q_t$</th>
<th>Hawkish on π</th>
<th>Strong mandate on output gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ Calvo price stickiness</td>
<td>0.75</td>
<td>0.1</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>β Subjective discount rate</td>
<td>0.996</td>
<td>0.996</td>
<td>0.996</td>
<td>0.996</td>
<td>0.996</td>
</tr>
<tr>
<td>ω Partial indexation</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$ν_y$ Output gap</td>
<td>0.125</td>
<td>0.125</td>
<td>0.125</td>
<td>0.125</td>
<td>0.5</td>
</tr>
<tr>
<td>$ν_q$ Real exchange rate</td>
<td>0.13</td>
<td>0.13</td>
<td>1</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>$ν_π$ Inflation</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>5</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Coefficients of

| $Δs_t$                           | 0.940    | 0.111          | 0.671            | 0.940        | 0.984                       |
| $π_t^f - π_{t-1}$                 | 0.342    | 0.053          | 0.244            | 0.342        | 0.358                       |
| $π_t^f - π_{t+1}$                 | 0.682    | 0.105          | 0.486            | 0.682        | 0.714                       |
| $π_t - π_t^f$                     | 0.693    | 10.260         | 0.494            | 2.309        | 0.181                       |
| $i_t - i_t^f$                     | 0.462    | 6.840          | 0.329            | 0.462        | 0.121                       |

IV.4 Explaining nonstationary real exchange rate

Can our model explain why the unit root null hypothesis for the case of Korea, for instance, cannot be rejected even when breaks are accounted for? The answer lies in Eq. (29). As real exchange rate is described as unit-root process if $ρ ≥ 1$, we get

$$\frac{(1-ρ_2)ν_y}{ν_y + ρ_2ν_q} ≥ 1$$

which simplifies to

$$ρ_2(ν_y + ν_q) ≤ 0 \text{, or}$$

$$ν_q ≤ −ν_y$$

as $ρ_2 > 0$. Eq. (32) argues that central banks that do not respond to change in the level of real exchange rate, $ν_q = 0$, real exchange rate can be characterized by unit root process if $ν_y ≤ 0$. 
The intuition is straightforward, particular for inflation targeters like Korea. Note that \( \hat{y}_t \) is log-deviation of actual output from steady state, which in practice implies Holdrick-Prescott filtered output. We can rewrite Eq. (24) in terms of output gap
\[
i_t = \beta^{-1} - 1 + (1 - \nu_y)\pi_\tau + \nu_y \bar{y}_t + \nu_q \hat{q}_t + \nu_{\Delta q} \Delta q_t + \epsilon_t
\]  
(33)
where \( \bar{y}_t = \hat{y}_t - \hat{y}_t^n \), \( \epsilon_t = \nu_y \hat{y}_t^n \), and \( \hat{y}_t^n \) denotes the log deviation of natural output from its steady state. Suppose there is a productivity advance that increases the natural output over the steady state, and the variance of the productivity shock \( \epsilon_t \) is increasing in the absolute value of \( \nu_y \). On the one hand, prices have tendency to fall in the aftermath of favorable productivity shock, unless aggregate demand is able to catch up with the rise of aggregate supply. On the other hand, what the market needs in such scenario is exactly an environment conducive to increasing demand to clear the market. Hence, optimal monetary response for inflation targeters like Korea should call for a contemporaneous fall in interest rate to accommodate the rise in natural output. To do so, inflation targeters need \( \nu_y < 0 \).

Put in our context, a fall in nominal and thus real interest rate below long-run average, given the sticky-price environment, of course causes a level-break in real exchange rates. According to Eq. (18), contemporaneous depreciated level of nominal exchange rate will increase the prospective risk premium of the currency, which, in turn, results in a level of nominal and thus real exchange rates that can be persistently above the long-run average. More successful is the central bank in stabilizing inflation fluctuation, more persistent the real exchange rates will be.

In short, the conduct of optimal monetary policy for inflation targeters can potentially be the culprit of unit root behavior in real exchange rates, despite the consideration of endogenous breaks.
V. Conclusion

This paper reconsiders real exchange rates stationarity in the framework of the minimum Lagrangian Multiplier unit root test that can account for two endogenously determined structural breaks. Free of criticism of arbitrary in break determination and the explicit consideration of breaks under unit root null have been the two main pillars that make this procedure the most appropriate one in testing for real exchange rate stationarity.

Our results are mixed. There are countries which exhibit trend-stationary real exchange rates, and there are the otherwise. This motivates us to address three important questions: what are exactly the driving forces of unit root process? What are the variables that constitute structural breaks? What are the factors that propagate or moderate the role of the found variables as source of break?

By shedding lights on a simple New Keynesian model, we show that the conduct of optimal monetary policy, which calls for an accommodating contemporaneous response to natural output fluctuation, can result in nonstationary real exchange rates, particularly when the currency risk premium is increasing in the level of nominal exchange rates. When a country’s real interest rate falls (rises) in response to productivity shock, its currency depreciates (appreciates) not only because home assets are lower (higher) yielding but also because the currency becomes more (less) riskier.

In addition, we conjecture that interest rate differential is the source of level-break, whereas the rate of nominal depreciation and inflation differentials can instigate mean-break. The magnitude of each factor is largely influenced by the degree of price stickiness and central bank’s preference toward inflation, output and real exchange rate stabilization.
What is particularly important, but is out of the scope of this paper, and thus is of interest for future study, is certainly the quantitative evaluation of the model of real exchange rate nonstationarity based on individual country experience. Because persistent deviation of real exchange rate from long-run average can end with crisis-provoking resource misallocation, probing into the relationship between optimal monetary policy and real exchange rate nonstationarity shall be a promising venue for future study.

References


