UECM3033 Numerical Methods

Tutorial : Solutions of Equations

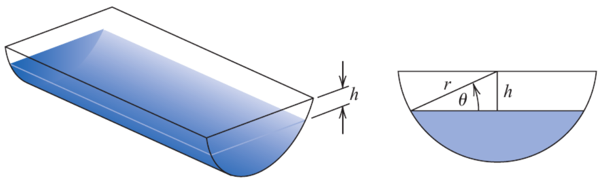
(Jan 2013)

1. Show that the equation $x^3 + 2x - 6 = 0$ has a root in $(1, 2)$. Use bisection method to approximate the root accurate up to 2 significant digits.

2. Let c be a constant and $g(x) = x + c(x^3 - 2)$. Consider the iteration $x_{n+1} = g(x_n)$.
   a) Find the limit of the iteration if it converges at all.
   b) Determine a value of c so that the iteration converges.

3. Keplers equation $x = m + E\sin(x)$ where m and E are constants, plays an important role in astronomy. Let $m = 0.8, E = 0.2, x_0 = m$.
   a) Show that it may be solved by using direct iteration $x_{n+1} = m + E\sin(x)$. Find the first three iterations.
   b) Find also the fist three approximations by using Newtons Method.

4. A trough of length L has a cross section in the shape of a semicircle with radius r. (See figure.) When filled with water to within a distance h from the top, the volume V of water is $V = L[0.5\pi r^2 - r^2 \sin^{-1}(h/r) - h\sqrt{r^2 - h^2}]$. Suppose $L = 10\text{cm}, r = 1\text{cm}$ and $V = 12.4\text{cm}^3$. Find the depth of water in the trough to within 0.01cm



5. The *reciprocal* of a number R can be computed without division by the iterative formula $x_{n+1} = x_n(2 - x_n R)$. Establish this relation by applying Newton's method to some $f(x)$. Beginning with $x_0 = 0.02$, compute the reciprocal of 17 correct to six decimal digits or more by this rule. Tabulate the error at each step and observe the quadratic convergence.

6. Apply the secant method on the $f(x)$ found in the previous question to evaluate the reciprocal for 17 correct to six decimal digits. Use the initial conditions $x_0 = 0.01$ and $x_1 = 0.02$.

7. Apply the fixed-point iteration method, starting with $x_0 = 3.1$, generate the sequence $\{x_n\}_{n=1}^{10}$ that approximates the solution for $x = g(x) = \ln(x) + 2$.
   a) Show that $\{x_n\}$ converges linearly to $x = 3.1419322$.
   b) Apply the Aitken's method to $\{x_n\}_{n=1}^{10}$ to speed up the convergence.
   c) Apply the Steffensen's method to find $\tilde{x}_i, i = 0, 1, 2$.