UECM3033 Numerical Methods

Tutorial : Computational Linear Algebra

(Jan 2013)

1. Let $\mathbf{A}_1 = \begin{bmatrix} 4 & -1 & 1 \\ -1 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and $\mathbf{A}_2 = \begin{bmatrix} 164 & 47 & 5 & 18 & 26 \\ 47 & 140 & 13 & 26 & 34 \\ 5 & 13 & 156 & 39 & 47 \\ 18 & 26 & 39 & 172 & 5 \\ 26 & 34 & 47 & 5 & 148 \end{bmatrix}$

   a) Detemine if $\mathbf{A}_1$ and $\mathbf{A}_2$ are (i) symmetric, (ii) strictly diagonally dominant, (iii) positive definite.

   b) Explain if you were using a Gaussian elimination to solve the linear system $\mathbf{Ax} = \mathbf{b}$, do you need to perfom partial pivoting? Why?

   c) Explain if iterative methods, such as Jacobi or Gauss-Seidel methods, will converge to the solution $\mathbf{x}$?

2. Solve the linear system $\mathbf{A}_1\mathbf{x} = \mathbf{b}_1$ and $\mathbf{A}_2\mathbf{x} = \mathbf{b}_2$, for $\mathbf{A}_1$ and $\mathbf{A}_2$ in Question 1, while $\mathbf{b}_1 = [-1, 8, 3]^T$ and $\mathbf{b}_2 = [1, 2, 3, 4, 5]^T$ by using

   a) $\mathbf{LU}$ factorisation.

   b) $\mathbf{LDL}^T$ factorisation.

   c) $\mathbf{LL}^T$ factorisation.

   d) Jacobi method

   e) Gauss-Seidel method

   f) (Optional*) Conjugate gradient method

3. Let $\mathbf{C}$ be the matrix $\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$.

   a) Use MATLAB and find the eigenvalues and eigenvectors of $\mathbf{C}$.

   b) Find also the algebraic and geometric multiplicity of each eigenvalue.

   c) Is $\mathbf{C}$ a defective matrix?

   d) Write down the Schur decomposition of $\mathbf{C}$.

4. Let $\mathbf{C}$ be the matrix in Question 3.

   a) Draw the Geršgorin circles for $\mathbf{C}$.

   b) Find the largest eigenvalue of $\mathbf{C}$ by power methods. (Carry out 10 iterations)

   c) Use Aitken's Δ^2 to improve the estimates for the largest eigenvalue.

   d) Find the other eigenvalues by using Inverse Power Method

e) (Optional*) Rayleigh Quotient method.

5. By using MATLAB, repeat Question 3 and 4 for the matrix

$$\begin{bmatrix} 2.2454 & -0.6969 & 2.5569 & -1.6969 & 0.1454 \\ 0.3113 & 0.9719 & 2.9585 & -1.5781 & -0.0637 \\ 0.2953 & -0.0054 & 5.0985 & -2.1054 & -0.0214 \\ 0.1724 & -0.6038 & 0.9538 & 2.4962 & -0.7109 \\ 0.1381 & -0.4408 & 1.5323 & -1.1408 & 2.1881 \end{bmatrix}.$$