UECM3033 Numerical Methods

Tutorial : Interpolation

(Jan 2013)

1. Given the following data:

x	- 0.75	-0.5	-0.25	0
$f(x)$	-0.07181250	-0.02475000	0.33493750	1.10100000

   a) Write down the Lagrange interpolating polynomial and estimate $f(-1/3)$.

   b) Use Newton's forward difference formula to construct interpolating polynomial and approximate the value of $f(-1/3)$.

   c) Use Newton's backward difference formula to construct interpolating polynomial and approximate the value of $f(-1/3)$.

2. Given the following data:

x	0.6	0.7	0.8	1.0
$f(x)$	-0.17694460	0.01375227	0.22363362	0.65809197

   a) Write down the Lagrange interpolating polynomial and estimate $f(0.9)$.

   b) Use Newton's forward difference formula to construct interpolating polynomial and approximate the value of $f(0.9)$.

   c) Use Newton's backward difference formula to construct interpolating polynomial and approximate the value of $f(0.9)$.

3. For the given functions $f(x)$, let $x_0 = 0, x_1 = 0.3, x_2 = 0.6, x_3 = 0.9$. Construct the Lagrange interpolating polynomials of degree at most 3 to (i) approximate $f(0.45)$, (ii) find the maximum error bound, and (iii) find the actual error.

   a) $f(x) = \cos x$

   b) $f(x) = \ln(x + 1)$

   c) $f(x) = \sqrt{1 + x}$

4. Consider the knots $A(0, 0), B(1, 1), C(6, 1)$ and $D(5, 2)$ over the partition $0 < 1 < 5 < 6$.

   a) Find a spline of degree 1 that interpolates the knots.

   b) Find a spline of degree 2 that interpolates the knots, and zero derivative at point A.

   c) Find a cubic spline that interpolates the first two knots, given the tangent vectors (i.e. the derivative of the spline) at knots A and B are 0 and $\frac{1}{4}$.

5. A natural cubic spline S on $[0, 2]$ is defnined by:

$$S(x) = \begin{cases} S_0(x) = 1 + 2x - x^3, & \text{if } 0 \le x < 1, \\ S_1(x) = a + b(x-1) + c(x-1)^2 + d(x-1)^3, & \text{if } 1 \le x \le 2. \end{cases}$$

   Find a, b, c and d.

6. Consider the table of points below:

x	0	0.25	0.50	0.75	1.0
y	1	0.70711	0	-0.70711	-1

   a) Construct a natural cubic spline that pass through the data.

   b) Construct a Newton's interpolating polynomial that pass through the data.

   c) Given that the data are generated from $f(x) = \cos(\pi x)$, find the relative error for the estimates of $f(0.8)$ from the spline and Newton's polynomial.