UECM3033 Numerical Methods

Tutorial : Numerical ODE & Boundary Value Problem

(Jan 2013)

1. Solve the following boundary value problem

$$x''(t) = \frac{2t}{1+t^2}x'(t) - \frac{2}{1+t^2}x(t) + 1, \quad x(0) = 1.25, x(4) = -0.95$$

by Shooting method.

2. Repeat Q1 with

$$x'' = -2x' - 2x + e^{-t}, \quad x(0) = 0.6, x(4) = -0.1.$$

3. The van der Pol equation

$$y'' - \mu(y^2 - 1)y' + y = 0, \quad \mu > 0,$$

governs the flow of current in a vacuum tube with three internal elements. Let $\mu = 1/2, y(0) = 0$, and $y(2) = 1$. Approximate the solution $y(t)$ by using shooting method with step size $h = 0.5$.

4. Rewrite the equation $y''(x) - xy'(x) + 4y(x) = x^2, y(0) = 0, y(6) = 0$ as a system of first order ODEs and construct a MATLAB function file called `dy.m` which defines the right hand side of the system and which is suitable for use with the `ode45` integration routine. Explain then how you could solve this problem using MATLAB.

5. The vibrating string equation has the form $y'' + \lambda^2 y = 0$ with λ unknown and subject to: $y(0) = 0, y(1) = 0$. It has a solution only for certain values of λ known as eigenvalue. Find λ and solve the equation by shooting method. [Hint: let $y_1 = y, y_2 = y', y_3 = \lambda$ and apply the fact that the ODE remains unchanged if we change $y \to ky$, for any k.]

6. Solve the following boundary value problem

$$x''(t) = 2x' - x + t^2 - 1, x(0) = 5, x(1) = 10$$

by finite-difference method with step size $h = 0.25$. Calculate the error at each mesh points, given that the exact solution is $x(t) = t^2 + 4t + 5$.

7. Repeat Q6 with $y'' = 4(y - x), \quad y(0) = 0, y(1) = 2$, and given the exact solution is $y(t) = e^2(e^4 - 1)^{-1}(e^{2x} - e^{-2x}) + x$.