4.0 Introduction

There are a few important applications for diode, which depend on how the diode is biased. Figure 4.1 shows different quadrants of the I-V characteristic used for six diode types, which are rectifying diode, light emitting diode, laser diode, the zener diode, photodetector, and solar cell.

Rectifying diode operates in quadrant one and three as shown in Fig. 4.1(a). Light emitting diode and laser diode operate in quadrant one as shown in Fig. 4.1(b). Zener diode and photodetector operates in quadrant three shown in Fig. 4.1(c). Lastly, solar cell operates in quadrant four as shown in Fig. 4.1(d).

![Figure 4.1: The operating quadrants of important diode types](image-url)
Emphasis is put on the operation of light-emitter diode at quadrant one of the I-V characteristic curve.

In this Chapter, we will discuss the principle of radiative recombination of hole and electron, the material used for building LED, and the construction of LED etc. If it is required, the review of fundamental theory for a better explanation of the text will be discussed.

4.2 A General Overview of Light Emitting Diode

Light emitting diode LED utilizes the principle of recombination of majority carrier in pn junction to produce light, which is also termed as injection electroluminescence. This will happen when \( pn > n_i^2 \). Forward biasing pn junction would inject majority carrier from each side of the p and n materials across the junction, whereby it will recombine with majority carrier at the other side of junction to produce visible light. The illustration is shown in Fig. 4.2.

![Figure 4.2: Injection of minority carrier and subsequent radiative recombination with majority carrier in a forward bias pn junction](image)

In the double heterojunction LED like \( p\text{AlGaAs}-i\text{GaAs}-n\text{AlGaAs} \), whereby a narrow energy band-gap material \( i\text{GaAs} \) is sandwiched between two wide energy band-gap materials like \( \text{AlGaAs} \), higher carrier density and improved carrier confinement due to higher refractive index, can be achieved as illustrated in Fig. 4.3. Moreover, due to the fact that the radiative recombination lifetime is shorten, it leads to more-efficient radiative recombination.
The injected carrier across the junction would increase the minority carrier at each side of the $p$ and $n$ materials. The injected carrier shall follow the equation (4.1).

$$\Delta n(x) = \Delta n(0) \exp(-x/L_n)$$

$L_n$ is the diffusion length of electron and $\Delta n(x)$ is the change of minority concentration $x$ distance away from the edge of depletion region. Similarly, the minority hole in $n$ material side would follow equation $\Delta p(x) = \Delta p(0) \exp(-x/L_p)$.

Figure 4.4 illustrates the steady-state minority carrier distribution of $pn$ junction under forward bias.

Indirect semiconductor such as silicon and germanium has very small probability of radiative transition due to lattice interaction basically phonon and other scattering agent that causing low radiative transition. We shall explain the reason using the concept of total conservation of wavevector. However, in today’s technology, indirect semiconductor can be doped or diffused with element such as nitrogen N and bismuth Bi providing capturing centre in the forbidden gap.

Direct semiconductor such as gallium arsenide, gallium arsenide phosphide $\text{GaAs}_{1-x}\text{P}_x$ or gallium indium arsenide phosphide recombines very efficiently and produces photon, which has an energy equal in magnitude to the transition energy. By proper mixing gallium indium arsenide phosphide compound semiconductor, LED can produce visible light range from infrared to violet.
Figure 4.4: The steady-state minority carrier distribution of pn junction under forward bias

$n_p(-W_p) = n_{po} \exp\left(\frac{eV_R}{kT}\right)$

Forward bias diffusion of electrons

$n_p(x)$

$n_{po}$

$-W_p$

$0$

$W_n$

$p^+$

$n$

$p_n(W_n) = p_{no} \exp\left(\frac{eV_R}{kT}\right)$

Forward bias diffusion of holes

$p_n(x)$

$p_{no}$

$x$

Figure 4.5 shows the energy band-gap of some semiconductors and their relative optical spectrum. The photon energy can be from 3.6eV ultraviolet - ZnS to 0.18eV infrared - InSb.

Indeed utilizing ternary such as $\text{Al}_x\text{Ga}_{1-x}\text{As}$ and $\text{GaAs}_{1-x}\text{P}_x$, and quaternary $\text{Ga}_x\text{In}_{1-x}\text{As}_{1-y}\text{P}_y$ semiconductor alloy and compound, the number of available energies can be increased significantly. The energy band-gap of various alloy or
compound can be approximated from *Virtual Crystal Approximation Method* using relationship

\[
E_G^{\text{Alloy}} = xE_G^A + (1-x)E_G^B 
\]  

(4.2)

where \(E_G^A\) is the energy band-gap of material A and \(E_G^B\) is the energy band-gap of material B respectively. However, there is bending factor C arising from the increasing disorder due to alloy. Thus, energy band-gap of alloy semiconductor is generally followed expression

\[
E_G^{\text{Alloy}} = a + bx + Cx^2 
\]  

(4.3)

As it is illustrated in Fig. 4.6, by varying the percentage of phosphorus and with arsenic, the energy band-gap can be varied from direct GaAs of 1.43eV (infrared LED) to indirect GaP of 2.26eV (green LED) as shown in Fig. 4.7.

![Figure 4.6](image)

**Figure 4.6:** Energy band-gap of GaAs\(_{1-x}\)P\(_x\) semiconductor alloy after considering the bowing factor. The formula used is \(E_G^{\text{Alloy}} = 1.424 + 1.150x + 0.176x^2\)
The formula to follow is $E_G^{\text{Alloy}} = 1.424 + 1.150x + 0.176x^2$. For $x$ value greater than 0.45, GaAs$_{1-x}$P$_x$ becomes an indirect semiconductor, whereby its probability of radiative transition is very low. A special *isoeletronic* recombination centers such as adding nitrogen to replace some phosphorus atoms would create the trapping center near the conduction band edge. Thus, it improves its radiative transition. We also termed it as *quantum efficiency*. Without nitrogen capturing centre the quantum efficiency is as low as 5% for Al$_x$Ga$_{1-x}$As with $x$ value greater than 0.45. With nitrogen as a isoelectronic combination center, the quantum efficiency is as high as 100%. Thus, the recipe to turn indirect semiconductor into high quantum efficiency diode is by mean of creating the capturing center in the forbidden band-gap of the diode. We shall discuss the reason why indirect semiconductor has low radiative transition later in the text.

![Figure 4.7: Energy band-gap of GaAs$_{1-x}$P$_x$ semiconductor alloy for $x$= 0 to 1.0](image)

### 4.3 Radiative Recombination

Radiative recombination in semiconductor occurs predominantly via three different processes namely interband transition, recombination via impurity centre, and exciton recombination. Yet each type of radiative recombination is
dependent on the semiconductor material types used and there are a number of sub-types within each type of radiative recombination.

### 4.3.1 Interband Radiation

Interband recombination process via direct and indirect energy band-gap material is illustrated in Fig. 4.8. Figure 4.8(a) shows the $E$-$k$ energy band diagram of silicon, which is an indirect semiconductor and gallium arsenide, a direct semiconductor.

![Figure 4.8: E-k energy bands diagram of silicon (indirect semiconductor) and gallium arsenide (direct semiconductor)](image)

The key thing for this transition is that the total wavevector must be conserved. The photon wavevector follows de Broglie principle is equal to $2\pi/\lambda$, while the electron wavevector is range between $-\pi/a$ and $\pi/a$, where $a$ is the lattice constant. This is also equal to $k$ values at the first Brillouin zone bounded region. The wavelength of the visible light region is 400nm to 800nm region, whereas the lattice constant of the semiconductor is a few Armstrong. This shall mean the wavevector of the photon is very much smaller than the possible magnitude of the wavevector of electron. Consequently, if the recombination involved electron as particle and photon, there total wavevector is not conserved. In order to conserve the total wavevector, it must involve virtually same wavevector, which is direct transition. i.e. direct semiconductor. Indirect transition is only happened if there is conservation of wavevector. Put in another
word, there must have creation or annihilation of phonon. Thus, the equation for wavelength of emitted photon is then given by equation (4.4).

\[ \frac{hc}{\lambda} = E_G \pm E_p \] (4.4)

The + and – sign denotes to phonon annihilation or creation respectively. The phonon energy \( E_p \) is usually in the order of 0.01eV. \( E_G \) is interband gap energy. Since now, it involves three particles namely electron, photon, and phonon, the transition is much less probable than two particle direct transition. The recombination in indirect semiconductor material is at time name as non radiative recombination since there is no release of photon.

In practice, electron and hole reside slightly above and below their energy band edges \( E_C \) and \( E_V \) respectively. The peak wavelength of the interband transition for the direct semiconductor material follows equation (4.5).

\[ \lambda = \frac{hc}{E_G + 1.8(kT)} \] (4.5)

1.8kT is the half-power width of small energy spread due to density of state in the conduction band or valence band, which is defined as \( N(E) = \frac{(2m^*)^{3/2}}{2\pi^3} \sqrt{E - E_G} \) and carrier distribution governed by Boltzmann distribution, which is \( F(E) = \exp\left(-\frac{E}{kT}\right) \). Thus, the spontaneous emission rate \( I(E= h\nu) \) is proportional to the product of these two equations, which is

\[ I(E= h\nu) \propto \sqrt{E - E_G} \exp\left(-\frac{E}{kT}\right) \] (4.6)

The plot of equation (4.6) shown in Fig. 4.9 shows that the peak of spectrum of spontaneous emission is \( (E_G + 1/2kT) \) and the half-power width is 1.8kT. This translates into a spectrum width in wavelength of

\[ \Delta\lambda \approx \frac{1.8kT\lambda^2}{hc} \] (4.7)

which is approximately 10nm width at middle of visible spectrum.
At any temperature the probability electron recombining is proportional to the number of hole present. Thus, the rate of electron recombination with hole is dependent on the product of concentration of hole and electron. The rate of interband recombination $R$ is shown in equation (4.8).

$$R = Bnp$$ (4.8)

$n$ and $p$ are concentration of electron and hole and $B$ is constant of proportionality. It is found that the values of $B$ for indirect semiconductor materials are around $10^6$ times smaller than for direct semiconductor. Take for example, the $B$ values for silicon Si, germanium Ge, gallium phosphide GaP are $1.79 \times 10^{-21}$, $5.25 \times 10^{-20}$, and $5.37 \times 10^{-20} \text{ m}^3 \text{s}^{-1}$ respectively and the $B$ values for gallium arsenide GaAs, indium phosphide InP, and indium arsenide InAs are $7.21 \times 10^{-16}$, $8.50 \times 10^{-17}$, and $4.58 \times 10^{-17} \text{ m}^3 \text{s}^{-1}$ respectively.

### 4.3.2 Impurity Center Recombination

There are basically three types of impurity center recombination. They are illustrated in Fig. 4.10. Figure 4.10(a) illustrates the conduction band to acceptor level recombination. Figure 4.10(b) illustrates donor level to valence band recombination, and Fig. 4.10(c) illustrates donor level to acceptance level recombination. However, the third types of recombination is only possible if the donor and acceptor levels are closed together. Consequently, there are multiple emission line width radiations from impurity center recombination.
4.3.3 Exciton Recombination

Exciton states exist within the energy band gap of pure semiconductor material. Photon enters a semiconductor, exciting an electron from the valence band into the conduction band. The hole in the valence band attracts an electron by the Coulombic force. The exciton results forms the binding of the electron with its hole. As the result, the exciton has slightly less energy than the unbound electron and hole. The wavefunction of the bound state similar to Bohr-like state whereby an electron and a hole circle around their common center of gravity relative large distance. The electron and hole are relatively weakly bound and the exciton states are situated just below the bottom of the conduction band. Figure 4.11 illustrates the model of an exciton. The bind energy $E_e$ of exciton is governed by equation (4.9).

$$E_e = 13.6 \frac{m^*_e}{m} \left( \frac{1}{\varepsilon_r} \right)^2 \text{eV} \quad (4.9)$$

$m^*_i$ is the reduced mass, which dependent on the effective mass of electron $m^*_e$ and hole $m^*_h$ using equation $\frac{1}{m^*_i} = \frac{1}{m^*_e} + \frac{1}{m^*_h}$. $m^*_i$ is equal to $m$ when effective mass of hole and effective mass of electron are equal to their rest mass in vacuum respectively.

For an example, GaAs has relative permittivity $\varepsilon_r = 11.5$, $m^*_e = 0.068m$, and $m^*_h = 0.47m$. Thus, reduced mass is $m^*_i = 0.06m$ and bind energy $E_e$ of exciton is 5.9meV. The value of $m$ is $\left[ \frac{1}{m_e} + \frac{1}{m_h} \right]^{-1} = \left[ \frac{1}{9.109 \times 10^{-31}} + \frac{1}{1.673 \times 10^{-27}} \right]^{-1} = 9.104 \times 10^{-31} \text{kg}$. 

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4.3.4 Emission Line

At a finite temperature electron in conduction band of a semiconductor are distributed amongst the energy levels with a probability of occupancy by Fermi distribution. The halfwidth of the distribution has energy of about 2kT (actually it is 1.8kT). Thus, the expected range of emission frequencies $\Delta \nu$ is $\frac{2kT}{h}$. This converted to an equivalent wavelength spread $\Delta \lambda_0$ in which mathematically can be expressed from the equations stated below. The frequency of the radiation is $\nu = \frac{c}{\lambda_0}$. The change of frequency $\nu$ with respect to wavelength $\lambda_0$ shall be

$$\frac{d\nu}{d\lambda_0} = -\frac{c}{\lambda_0^2}$$

(4.10)

Hence the change of wavelength is $\Delta \lambda_0 = \frac{2kT\lambda_0^2}{hc}$. This would contribute to emission line widths.

A wavelength of 900nm at temperature 300K, the predicted half-width wavelength of 38nm is agreeable with the practical observation.

4.4 LED Materials

The requirements for a suitable LED material design are: It must have an energy band-gap of appropriate width. Secondly, both $p$- and $n$-types must exist preferably with low resistivities. Finally, the efficient radiative pathways must be present. In order to obtain visible radiation, the energy band gap of the

Figure 4.11: Model of an exciton where hole and electron may be regarded as being bound together and orbiting around their common center of gravity
materials have to be at least or greater than 2.0eV. Material that has large energy band-gap tends to have high resistive even when it is doped. Large band-gap material normally is difficult to prepare because of high melting temperature and low structural stability.

There are many commercial available LED materials. We shall discuss some of them here.

Gallium arsenide GaAs is a direct band gap material with energy band-gap 1.43eV ($\lambda_G = 860$nm). The LED material can be done by diffusing zinc into $n$-type GaAs. The radiation recombination is band to band transition. However, due to re-absorption, it reduces the device efficiency and shifts the peak emission wavelength to about 870nm. More efficient diode may be made by using silicon as a dopant.

Gallium phosphide GaP is an indirect band-gap semiconductor with $E_G = 2.2$eV ($\lambda_G = 549$nm) and hence band to band transition is rare. Group V element such as nitrogen N, bismuth Bi, and ZnO can be used as dopant to assist radiative transition. These element replace the phosphorus atoms and form recombination center named as isoelectronic trap centers. This method improves the quantum efficiency of the transition. The isoelectronic trap level is about 8meV below the conduction level. Therefore, the peak wavelength of the emission is slightly less than $\lambda_G$. Using increased levels of nitrogen doping and also doping with zinc and oxygen simultaneously given rise to deeper traps and consequently higher emission wavelengths. The system can be viewed from $E$-$k$ diagram shown in Fig. 4.12. Such doping will not violate the conservation of momentum because the isoelectronic trap is highly localized in space and because of Uncertainty principle, it has wide range in the $k$-space momentum.

The isoelectronic center is normally neutral. The injected electron is first trapped at the center. The negatively charge enters then captures a hole from the valence band to form bound exciton. Subsequent annihilation of this electron-hole pair yields photon with energy equal to the energy band-gap minus an activation energy of the center.
The quantum efficiency for GaAs$_{1-x}$P$_x$ with nitrogen is considered high for $x > 0.5$ as shown in Fig. 4.13. But it steadily decreases with increasing $x$ value because of the increasing separation in momentum between direct and indirect band-gap.

Figure 4.12: $E$-$k$ diagram showing radiative recombination through isoelectronic trap in indirect energy band-gap

Figure 4.13: Quantum efficiency of GaAs$_{1-x}$P$_x$ with different composition with and without nitrogen trap center
The band-gap of ternary alloy such as GaAs$_{1-x}$P$_x$ depends on the x value. The value of x less than 0.45 would turn this ternary alloy into direct semiconductor. For value x > 0.45, the ternary semiconductor turns from direct semiconductor to indirect semiconductor.

Highly efficient red and near infrared emitting LED can be made from gallium aluminum arsenide Ga$_x$Al$_{1-x}$As. If a heterojunction is formed between n-type Ga$_{0.3}$Al$_{0.7}$As and p-type Ga$_{0.6}$Al$_{0.4}$As, electrons injected from n surface layer into p-material recombine radiatively via acceptor levels and result in radiation of 650nm wavelength. This light can pass through the surface layer with little attenuation because of the relative large band-gap of the latter.

Radiation from green color until infrared can be made from III-V nitrides such as gallium nitride GaN and aluminum nitride AlN. Owing to lack of suitable substrate that matches with lattice constant of the materials and thermal expansion coefficient, the problem was resolved by adding a buffer layer that deposition on sapphire silicon carbide.

High brightness LED is commercially available based on InGaAs. By increasing the indium content the wavelength increase and radiation in the green can be obtained with reasonable high efficiency.

II-VI semiconductor materials such as zinc selenide ZnSe and its related compound such as ZnTeSe are suitable and have been successfully used to make blue and green LED. Green LED has been made using ZnTeSe as an active region grown on a ZnSe substrate. By replacing ZnTeSe layer with ZnCdSe, emission has been obtained in blue color. However, these materials are much softer than III-V nitride materials and degrade more rapidly at elevated temperature and consequently have shorter working life time.

Silicon carbide has been a promising material for LED for many year due its high melting point and consequently growth difficulties. It can be doped as both n- and p-type and commercial blue-emitting LED in recent years. Doping with B, Al, Sc and Be allow yellow, blue, green, and red emission respectively.

### 4.5 Construction of Light Emitting Diode

A typical LED construction called surface emitter type is shown in Fig. 4.14 illustrating the structure designed based on GaAsP semiconductor. It has an active thin layer of a few micrometers grown on a substrate, which has approximately 200µm thickness. The p-type is usually fabricated with thin approximately 5.0µm as compared to n-type of 50µm to minimize absorption.
The obvious advantage is to enable radiative recombination to be taken place near from the side of junction near the surface. Another obvious reason is to reduce the chance of re-absorption. The fraction \( \eta_e \) of total current injected in the \( p \)-side, which is the electron follows equation (4.11).

\[
\eta_e = \frac{D_e n_p/L_e}{D_e n_p/L_e + D_p p_n/L_n} = \left(1 + \frac{D_p L_e p_n}{D_e L_p n_p} \right)^{-1}
\]  

(4.11)

\( D_e n_p/L_e \) is the electron current and \( D_p p_n/L_n \) is the hole current. If Einstein’s equation \( D_{p,e} = (kT/e)\mu_{p,e} \) and Law of Mass Action \( n_p N_A = N_D p_n = n_i^2 \) are used to replace the minority concentration and diffusion coefficient, then equation (4.12) shall become

\[
\eta_e = \left(1 + \frac{\mu_p L_e p_n}{\mu_e L_p n_p} \right)^{-1}
\]  

(4.12)

![Figure 4.14: Standard structure of surface emitter red LED based on GaAsP semiconductor](image)

From equation (4.12), once can see that in order to achieve high fraction of electron current near unity, the \( n \)-type should be doped with higher doping concentration than the concentration of \( p \)-type, which is \( N_D >> N_A \) beside with already high mobility \( \mu_e \) of electron than the mobility \( \mu_p \) of hole. We may assume that the diffusion length of hole \( L_p \) is approximately equal to the diffusion length of electron \( L_e \). Thus, the diode is designed with as \( pn^+ \) diode.

Another normally constructed LED is \( ege \) emitter type. Shown in Fig. 4.15 is a double heterojunction LED that has higher efficiency and it also serves as the
window to emitted radiation because the higher energy band-gap semiconductor
confining layer does not absorb radiation from lower energy band-gap emitter
region. The active layer and the double heterojunction is sandwiched by two
optical cladding layers, which act as waveguide. The radiation is more
collimated so it does not suffer from total reflection due to critical angle.

Figure 4.15: Edge emitter red LED based on InGaAsP semiconductor

4.6 Efficiency of LED

In this section, the different types of LED efficiencies are discussed. They are
internal quantum efficiency, external quantum efficiency, optical efficiency,
power efficiency, and luminous efficiency.

4.6.1 Internal Quantum Efficiency

For a given input power, the radiative recombination process is in direct
competition with non-radiative recombination. The internal quantum efficiency
\( \eta_{in} \) is the efficiency of converting carrier current into photon, which defined as

\[
\eta_{in} = \frac{\text{Number of photon emitted internally}}{\text{Number of carrier passing junction}}
\]

It can be related to the fraction of injected carrier that combines radiatively to
the total rate of recombination and also be related with lifetimes of radiative and
non-radiative photon, which is defined in equation (4.14).

\[
\eta_{in} = \frac{R_r}{R_r + R_{nr}} = \frac{\tau_r}{\tau_r + \tau_{nr}}
\]
where $R_r$ and $R_{nr}$ are the radiative and non-radiative rate of recombination, and $\tau_r$ and $\tau_{nr}$ are lifetimes of radiative and non-radiative recombination.

For low level injection, the rate of radiative recombination in $p$-side is given by

$$R_r = R_{ec} n p = R_{ec} \Delta n N_A$$  (4.15)

where $R_{ec}$ is the efficiency of recombination and $\Delta n$ is excess electron carrier density i.e. $\Delta n >> n_{po}$. $R_{ec}$ is a function energy band-gap structure and temperature. Its value is approximately equal to $10^{-10}$ cm$^3$/s for direct semiconductor and $10^{-15}$ cm$^3$/s for indirect semiconductor material.

For low level injection, whereby $\Delta n << p_{po}$, the lifetime of radiative recombination $\tau_r$ is equal to equation (4.16).

$$\tau_r = \frac{\Delta n}{R_r} = \frac{1}{R_{ec} N_A}$$  (4.16)

For high level injection, $\tau_r$ would decreases with increases $\Delta n$. Thus, in double heterojunction LED, carrier confinement increases $\Delta n$ and $\tau_r$ reduces to improve internal quantum efficiency.

The non-radiative lifetime $\tau_{nr}$ is usually attributed to trap density $N_t$ or recombination centers, in which is follow equation (4.17).

$$\tau_{nr} = \frac{1}{\sigma V_{th} N_t}$$  (4.17)

where $\sigma$ is the capture cross section and $V_{th}$ is the thermal voltage. It is evident that the radiative lifetime $\tau_r$ needs to be small to yield high internal quantum efficiency.

### 4.6.2 External Quantum Efficiency

For LED applications, light emitted to external from the device. Thus, the optics inside and outside the device has to be considered. The parameter to measure the efficiency of light out externally is the optical efficiency $\eta_{op}$ or at time it is called extraction efficiency. With this factored in, the net external quantum efficiency is defined as
The optical efficiency is a subject of optics inside and around the devices and is totally independent of electrical phenomena. Let’s discuss it in next sub-section.

4.6.3 Optical Efficiency

Although the internal quantum efficiency of some LED materials approaches 100%, the external efficiency is much lower. Optical loss accounts the reason. There are basically three types of losses that reduces the optical efficiency of LED. They are absorption with the LED material, Fresnel loss, critical angle loss.

The absorption loss of LED can large. LED constructed on GaAs substrate has large loss since the substrate is opaque to light and it absorbs about 85% of the photons emitted from the junction. For LED constructed on transparent substrate such as GaP with isoelectronic center, photons emitted downward can be reflected back with about 25% absorption. The efficiency can be significantly improved.

The Fresnel loss is due to internal reflection back to the semiconductor. For normal incident radiation, there is no change of angle. However, due to reflection, the Fresnel loss can be calculated from reflection coefficient which is defined as

\[ R = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2 \]  

The third loss mechanism is caused by the total internal reflection of photon incident to the surface at angle greater than the critical angle \( \theta_c \). The reason being most of the emitted radiation strike the material interface at angle greater than the critical angle and so it remains trapped. This is because the refractive indices of III-V materials are high. With the interface with air, which has low refractive index, it is easily exceeding the critical angle for total internal reflection. The illustration of phenomenon of total internal reflection is shown in Fig. 4.16. Beam 1 is able to enter into medium 2 with refractive index \( n_2 \) because the incident angle \( \theta \) at medium 1 with refractive index \( n_1 \) is less than the critical angle \( \theta_c \). Beam 2 enters with at the medium interface with incident angle equal to critical angle \( \theta_c \). The light is reflected and traveled parallel with the interface. Light 3 incidents with angle greater than the critical angle of total
reflection. The beam never enters into medium 2. This phenomenon can be explained using Snell’s law. Snell’s law states the critical angle for total reflection follows equation (4.19).

$$\theta_c = \sin^{-1}(n_2 / n_1)$$  \hspace{1cm} (4.20)

From Snell’s law, it is clearly shown that the critical angle $\theta_c$ is depending on ratio of $n_2/n_1$. If the $n_2$ is the refractive index of air, which is small and $n_1$ is the refractive index of III-V semiconductor, which is high, the critical angle $\theta_c$ is small.

Light originated from recombination center near the $pn$ junction is radiated isotropically, whereas only that within a cone of semi-angle $\theta_c$ will escape. It shows that the fraction of the total generated radiation that is actually transmitted into the medium 2. If one assumes that the radiation is at the center of a sphere, then the surface area $A_{\text{cone}}$ of a cone with solid angle $\theta_c$, after ignored the surface curvature, is approximately equal to

$$A_{\text{cone}} = \pi (R \sin \theta_c)^2$$  \hspace{1cm} (4.21)

where R is the distance from the source to the interface surface. Thus, optical efficiency $\eta_{\text{opt}}$ due to critical angle $\theta_c$ is equal to
If we now combine equation (4.19) and (4.22), the optical efficiency $\eta_{\text{op}}$ after ignoring absorption is equal to equation (4.23).

$$
\eta_{\text{op}} = \frac{\pi(R \sin \theta)^2}{4\pi R^2} = \frac{\sin^2 \theta e}{4} = \frac{1}{4} \left( \frac{n_2}{n_1} \right)^2
$$

(4.22)

Based equation (4.23), it seems that the optical efficiency of a typical plane surface semiconductor LED is less than 2%.

One interesting phenomenon observed from Snell's law is that even though light inside the semiconductor has uniform intensity, light emitted into the ambient after refraction at the interface is angle dependence. It has a maximum intensity when the light is normal to the interface then decreases when the angle $\theta_2$ increases. Based on this fact, equating the light energy below and above the interface, it can be shown that for a typical planar LED structure, the emitted light intensity has an angle dependence on equation (4.24).

$$
I_2(\theta_2) = \frac{P_s}{4\pi r^2} \left( \frac{n_2}{n_1} \right)^2 \cos \theta_2
$$

(4.24)

where $P_s$ is the power of the light source and $r$ the distance of the surface from the source. Such an emission pattern is called Lambertian emission pattern as shown in Fig. 4.17. Figure 4.17 shows the emission patterns for a planar structure, a hemispherical structure, and a parabolic structure. It can be seen that for a planar structure, at an angle of 60° the normalized intensity drops to 50%. For an ideal hemisphere, since all rays are normal to the interface, the emitted intensity maintains uniformly high and the critical angle loss is totally eliminated. However, in practice such a hemispherical shape is very hard to achieve. A good practical compromise is to cap the planar structure with a hemispherical coating material that has refractive index lies between the semiconductor and the ambient.
Based on the normalized Lambertian emission pattern, to increase the value of optical efficiency $\eta_{op}$ is ensured that the radiation incident at angle less than critical angle $\theta_c$. Beside avoiding designing the LED in planar structure, there are two common ways to achieve, which is shown in Fig. 4.18.

Figure 4.17: The normalized Lambertian emission pattern for plane, hemisphere, and parabolic LED structure

Figure 4.18 illustrates two methods used to reduce critical angle loss in LED. The method shown in Fig. 4.18(a) is to design the $p$-type material into a hemispherical dome shape so that the light will strike at the interface at angle less than the critical angle. The second method as mentioned earlier, is to cap the $pn$ junction with plastic encapsulation of higher refractive index than air so
that the loss at the plane between semiconductor/plastic interface is less than the loss at the corresponding semiconductor/air interface. Take for an example, GaAs has refractive index of 3.6, plastic has 1.5, and air has 1.0 formed a good interface to reduce loss due to critical angle.

4.6.4 Power Efficiency

The power efficiency $\eta_p$ is simply defined as the ratio of the light power output to the electrical power input, which is shown in equation (4.25).

$$\eta_p = \frac{\text{Optical output power}}{\text{Electrical input power}} = \frac{\text{Number of photon emitted external x hv}}{\text{Number of photon emitted external x hv}} \frac{\text{IxV}}{\text{Number of photon emitted external x hv}}$$

$$= \frac{\text{Number of carrier passing junction x eV}}{\text{Number of photon emitted external x hv}}$$

(4.25)

Since the bias voltage $V$ is approximately equal to the energy band-gap and light energy ($eV=\text{hv}$), it follows that the power efficiency is similar to the external quantum efficiency i.e. $\eta_{ex} = \eta_p$, which can be determined by equation (4.23) if we assume that the internal quantum efficiency is equal to 1.

4.6.5 Luminous Efficiency

When comparing the visual effects of LEDs, the eye response must also be taken into account. The luminous efficiency normalizes the power efficiency by a factor that is related to eye sensitivity as shown in Fig. 4.19. For example, the human eye has a peak sensitivity at 0.555 $\mu$m (green color). As the wavelength approaches the red end or violet end of the visible spectrum, the sensitivity falls rapidly. So it takes less power in green color than other colors to achieve the same visual brightness for human eyes. For LED applications in display and illumination, luminous efficiency is a more appropriate parameter. The brightness of light output is measured by the luminous flux (in lumens),
Figure 4.19: Visible and near-visible electromagnetic spectrum. The visible portion is expanded at the top, and divided into major color bands. Also indicated is relative luminosity function $V(\lambda)$ as defined by the Commission of Internationale de l’Eclairage CIE for normal photopic vision.

The brightness of light output is measured by the luminous flux (in lumens), which is defined as

$$\text{luminous flux} = L_o \int V(\lambda)P_{op}(\lambda)d\lambda \text{  \text{lm}} \tag{4.26}$$

where $L_o$ is a constant with a value of 683 lm/W, $V(\lambda)$ the relative eye sensitivity, and $P_{op}(\lambda)$ the power spectrum of the radiation output. The eye sensitivity function $V(\lambda)$ is normalized to unity for the peak at $\lambda=555\text{nm}$. The luminous efficiency is then given by

$$\eta_{lu} = \frac{\text{luminous flux}}{\text{Electrical power in}} = \frac{683 \int V(\lambda)P_{op}(\lambda)d\lambda}{VI} \text{  \text{lm/W}} \tag{4.27}$$

The maximum luminous efficiency has a value of 683lm/W.

As LED technology advances with time, the luminous efficiency has achieved impressive progress. The chronological improvement of the luminous efficiency is summarized in Fig. 4.20. The luminous efficiencies for
conventional lightings are also included for comparison. The slope in the figure shows an improvement of a factor of two for every 3 years, or equivalently tenfold per decade. Obviously such rate of improvement cannot be sustained as the luminous efficiency approaches the theoretical limit of 683lm/W. By now, the most advanced LEDs have luminous efficiencies already surpassing those of the traditional lighting.

Figure 4.20: Progression of LED luminous efficiency with time

4.7 Infrared LED

The surface-emitting infrared InGaAsP LED that emits light from 1,100nm to 1,600nm used for optical-fiber communication system is shown in Fig. 4.21. The light is emitted from the central surface area and coupled into optical fiber. Using InGaAsP/InP can increase the efficiency and would give confinement of carrier by the layers of the higher band-gap semiconductor InP surrounding the radiative-recombination region InGaAsP. This heterojunction device can also serve as an optical widow to the emitted radiation because the higher energy band-gap confining layer does not absorb radiation from the lower band-gap emitting region.

The electrical input signal is generally modulated at high frequencies. This signal causes direct modulation of the injected current in an LED. Parasitic elements such as the depletion capacitance and series resistance can cause a delay of carrier injection into the junction and a delay in light output. This shall
mean how fast one can vary the output is depending on the carrier lifetime, which is determined by the type of recombination processes. We shall discuss the response time of diode in later section.

If the current is modulated at an angular frequency $\omega$, the light output power $P(\omega)$ is given by equation (4.28).

$$P(\omega) = \frac{P(0)}{\sqrt{1 + (\omega \tau)^2}}$$

(4.28)

$P(0)$ is the light output without modulated frequency i.e. $\omega = 0$ and $\tau$ is the carrier lifetime. The modulation band-width $\Delta f$ is defined as the frequency at which the light output is reduced to $1/\sqrt{2}$ or -3dB at no frequency i.e. $\omega = 0$. Thus, the modulated band-width is defined as

$$\Delta f = \frac{\Delta \omega}{2\pi} = \frac{1}{2\pi \tau}$$

(4.29)

**Figure 4.21:** GaInAsP/InP surface-emitting LED structure

### 4.8 White LED

White light LEDs is for general-purpose high brightness illumination. This application is becoming more and more important as both the power efficiency and the brightness have been improved to the extent that it is in direct competition with conventional lightings, i.e. incandescent and fluorescent lighting.
White light can be produced by mixing two or three colors of an appropriate intensity ratio. There are basically two approaches to achieve white light. The first approach is to combine LEDs of different colors, which are red, green, and blue. This is not a popular approach since it is more costly, and mixing of multiple colors of narrow bandwidth does not produce good color rendering. The second approach, which is the most commonly used, is to have a single LED covered with a color converter. A color converter is a material that absorbs the original LED light and emits light of different frequency. The converter material can be phosphor, organic dye, or another semiconductor. Of the three materials, phosphor is the most common type. The light output from a phosphor generally has a much broader spectrum compared to the LED light, and the wavelength range is longer. The efficiencies of these color converters can be as high as near 100%. One popular way is to use a blue LED together with a yellow phosphor. In this way, the blue LED light is partially absorbed by phosphor. The blue LED light is mixed with yellow light produced by the phosphor to produce white light. Another method is to use a UV LED. The UV LED light is completely absorbed by phosphor, and a wide spectrum of light is reproduced that emulates white light.

4.9 Organic LED

So far we have discussed LED made from inorganic semiconductor such as GaAsP, GaN etc. There is another branch of LED made from organic semiconductor material which usefulness particularly in multicolor large area flat panel display because of its attributes of low power consumption and excellent emissive equality with side view angle. Figure 4.22(a) shows the molecular structure of two representative organic semiconductors. They are tri(8-hydroxy-quinolinato) aluminum (AlQ₃), which contains six benzene rings connected to a central aluminum atom and the aromatic diamine, which also contains six benzene rings. However, they are in different molecular arrangement.

A basic organic LED OLED is shown in Fig. 4.22(b). It has a number of layers sitting on the glass substrate. The anode is made from indium tin oxide ITO and the cathode is made from Mg/Ag alloy. The diamine acts as hole transport layer and the AlQ₃ acts as the electron transport layer.

The energy band diagram of the OLED is shown in Fig. 4.22(c). It is basically a heterojunction formed between AlQ₃ and diamine. Upon bias the electrons from cathode are injected into the heterojunction interface. At the same time the holes from anode are also injected into the interface. Owing the
energy barrier $\Delta E_c$ and $\Delta E_v$, the holes and electrons are trapped or accumulated that enhance the chance of radiative recombination to produce photon. The barrier potential $e\phi_1$ and $e\phi_2$ between the diamine and ITO and Mg/Ag and AlQ$_3$ must low enough to allow high injection for high current density operation.

The forward current-voltage characteristic of LED is similar to that of the $pn$ junction. At low forward voltage, the diode current is dominated by non-radiative recombination current due to mainly surface recombination near the perimeter of the LED integrated circuit. At higher forward voltage, the diode current is dominated by the radiative diffusion current. At much higher forward voltage, the series resistance will limit the diode current. The total diode current can be written as

$$I = I_d \exp \left( \frac{e(V_R - IR_S)}{kT} \right) + I_r \exp \left( \frac{e(V_R - IR_S)}{kT} \right)$$

(4.30)

$R_S$ is the series resistor and $I_d$ and $I_r$ are saturation currents due to diffusion and recombination respectively.

Figure 4.22: (a) Organic semiconductor, (b) OLED cross sectional view and (c) band diagram of an OLED

AlQ$_3$ organic semiconductor material emits green light. By choosing different organic semiconductor material with different band-gap, various color including red, yellow, and blue can be obtained.

4.10 Current of LED

The forward current-voltage characteristic of LED is similar to that of the $pn$ junction. At low forward voltage, the diode current is dominated by non-radiative recombination current due to mainly surface recombination near the perimeter of the LED integrated circuit. At higher forward voltage, the diode current is dominated by the radiative diffusion current. At much higher forward voltage, the series resistance will limit the diode current. The total diode current can be written as

$$I = I_d \exp \left( \frac{e(V_R - IR_S)}{kT} \right) + I_r \exp \left( \frac{e(V_R - IR_S)}{kT} \right)$$

(4.30)

$R_S$ is the series resistor and $I_d$ and $I_r$ are saturation currents due to diffusion and recombination respectively.
4.11 Response Time of LED

For display purpose, the fast response time of less than 1.0µs obtainable from LED is not essential. However, for optical communication purpose, it is important. There are two factors limit the response time of the diode. The junction capacitance $C_j$ arises from the variation of charge stored in the depletion region due to various applied external voltage. The second factor that limit the response time is the diffusion capacitance as the result of forward biasing the diode caused by storage of mobile carriers within the diffusion length. This extra charge requires extra time to diffuse and recombine with majority carrier in the diode. When the external voltage varies, the reverse recovery time $t_{rr}$ can be varied. The cut-off frequency $f_T$ of the LED is given by

$$f_T = \frac{1}{2\pi \tau}$$

(4.31)

where $\tau$ is the overall lifetime which is defined as

$$\frac{1}{\tau} = \frac{1}{\tau_r} + \frac{1}{\tau_{nr}}$$

(4.32)

The internal quantum efficiency is related to the radiative and non-radiative lifetimes $\tau_r$ and $\tau_{nr}$. If $\tau_r \ll \tau_{nr}$, then $\tau$ approaches $\tau_r$, as indicated in equation (4.15). $\tau_r$ decreases as the doping concentration in the active layer is increased and $f_T$ becomes larger. Therefore, for speed is concerned, increasing doping concentration in the middle active layer of the heterojunction is desired.

4.12 Applications of LED

Typical operating current of the LED is between 20mA to 100mA and forward voltage of the LED range from 1.2V for GaAs type to 2.0V for GaP type. There are many applications for LED. Listed here are some applications. The typical ac and dc circuit utilizing LED are shown in Fig. 4.23.

The resistor $R_s$ is used to limit the current flowing into the circuit, whilst the diode is connected to prevent excessive reverse voltage applied to the diode during the negative cycle of the ac voltage.
The typical modulation circuit utilizing LED is shown in Fig. 4.24. Circuit shown in Fig. 4.24(a) is a simple on/off modulation via a switch.
Circuit shown in Fig. 4.24(b) is a voltage divider bias modulation circuit whereby its output is modulated input voltage $V_{in}$.

An important application of infrared LED is opto-isolator, where an input or control signal is decoupled from output as shown in Fig. 4.25. The input signal causes the infrared LED to emit light that would be detected by photodiode and subsequently converted the input signal back to electrical signal.

Figure 4.25: An opto-isolator in which the input signal is decoupled from output signal

Another important application of infrared diode is for transmission of an optical signal through an optical fiber. An optical fiber is a wave guide at optical frequencies. The fiber is usually drawn from a preform of glass to a diameter of about 100µm. It is flexible and can guide optical signal over distance of many kilometers to receiver.

There are two types of optical fibers which are shown in Fig. 4.26. They are step-index fiber and graded-index fiber. The step index fiber has a cladding layer of relatively pure fused silica SiO$_2$ surrounding a core of doped glass such as germanium doped glass of high refractive index. The light is transmitted along the length of the fiber by internal reflection. Note for this type of fiber, different light transmit different path. The step index fiber will end with pulse spread at the end of transmission in the fiber.

The grade index fiber shown in Fig. 4.26(b) has the decreases from the core center by parabolic law. The light traversing toward the cladding has a high velocity due to lower refractive index than the ray along the center of the core. As the result less light spread occurs.
The basic elements of an optical fiber transmission link are shown in Fig. 4.27. It consists of a transmitter that comprises of driving circuit and LED or laser, optical fiber and a receiving consists of photodetector, amplifier and signal restorer.
Exercises

4.1. Why the rate of radiative recombination is very low for indirect semiconductor?

4.2. How scientist can make indirect semiconductor to have high radiative recombination?

4.3. The spectrum for spontaneous emission is given by equation

\[ I(E = \hbar \nu) \propto \sqrt{\frac{E - E_G}{kT}} \exp\left(-\frac{E}{kT}\right). \]

Find the photon energy at peak of the spectrum and the spectrum width.

4.4. GaAs has energy band-gap of 1.43eV. What is the wavelength of radiative recombination assuming inter-band recombination is occurring?

4.5. A light emitter diode is made from GaAs\(_{1-x}\)P\(_x\) semiconductor alloy. What should be the maximum value of x GaAs\(_{1-x}\)P\(_x\) to maintain as direct semiconductor? For this value of x what is the color of the illuminated LED?

4.6. What should be the value of x for a light emitter diode that made from GaAs\(_{1-x}\)P\(_x\) semiconductor alloy to emit red light?

4.7. If the electron current of a GaAs LED of depletion thickness 1.2\(\mu\)m is 20 times its hole current, what is the value of its injection efficiency?

4.8. III-V semiconductor generally high refractive index, discuss how the LED should constructed to achieve high external quantum efficiency.

4.9. Calculate the electrical-to-optical efficiency of an Al\(_{0.3}\)Ga\(_{0.7}\)As LED operated at 0.898\(\mu\)m.

4.10. The GaAs/plastic/air interfaces have refractive indices 3.6, 1.5, and 1.0 respectively. Calculate the fraction optical efficiency of total radiation generated actually transmitted into air.

4.11. Assuming that the radiative lifetime \(\tau_r\) is 10\(^9\)/N second, where N is the doping concentration of semiconductor in cm\(^{-3}\) and the non-radiative lifetime \(\tau_{nr}\) is 10\(^7\)s. Find the cut-off frequency of LED having doping concentration 10\(^{19}\) cm\(^{-3}\).
4.12. Calculate the modulated bandwidth of a GaAs-based LED that has carrier lifetime equals to 500ps.

4.13. Draw the modulated output signal of the modulation circuit shown in Fig. 4.25(b).