Chapter 7

Solar Cell

7.0 Introduction

Solar cells are useful for both space and terrestrial application. Solar cells furnish the long duration power supply for satellites. It converts sunlight directly to electricity with good conversion efficiency. It is a form of green energy.

7.1 Solar Radiation

The radiative energy output from the sun derives from nuclear fusion reaction. In every second about $6.0 \times 10^{11}$ kg of hydrogen is converted into helium with the net mass loss of about $4 \times 10^3$ kg. The mass loss is converted into energy using Einstein’s equation $E = mc^2$ that equates to $4.0 \times 10^{21}$ J. This energy is emitted primarily as electromagnetic radiation in the ultraviolet to infrared region that has wavelength ranges from 0.2 $\mu$m to 3 $\mu$m. The total mass of sun is now about $2.0 \times 10^{30}$ kg which can provide constant energy output for over next 10 billion years. Looking at color of the sun, the sun has gone through half of its life time.

The intensity of the solar radiation outside the earth atmosphere measured at the average distance of the earth orbit around the sun is $1367$ W/m$^2$. This value is also called solar constant. In the terrestrial surface the intensity of the solar energy is attenuated due to scattering of the cloud and atmosphere. The attenuation depends on the length of the light’s path through atmosphere or air mass, which is defined as the $1/cos\phi$, where $\phi$ is the angle between vertical line and the sun’s position. The air mass can be estimated from the height $h$ of the vertical object and the length $s$ of the shadow of this object. This equation to calculate the air mass is

$$\text{Air mass (AM)} = \sqrt{1 + \left(\frac{s}{h}\right)^2} \quad (7.1)$$

Figure 7.1 shows the two solar spectral irradiances (power per unit area per unit wavelength) curves. The one with AM equals to 0 zero is applicable for satellite operating in space and the one with AM equals to 1.5 is specified as reference for terrestrial solar cell application. This is spectrum representing sunlight at the
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earth’s surface when it 48° from the vertical line. Showing in the spectral curve is the cutoff wavelengths of silicon and gallium arsenide.

![Spectral Irradiance Graph]

**Figure 7.1:** The spectral irradiance of solar radiation for air mass 0 and 1.5 respectively showing the cutoff wavelengths of silicon and gallium arsenide

### 7.2 pn Junction Solar Cell

A solar cell is a *pn* junction with no voltage directly applied across the junction. The solar cell converts photon power into electrical power by generation of electron-hole pair EHP and delivers this power to a load. A typical silicon solar cell is shown in Fig. 7.2. Power is generated from solar cell since it operates in forth quadrant of the I-V characteristic curve of the diode. Only photons have the energy $h\nu$ greater than the energy band-gap $E_G$ of the solar cell material, electron-hole pairs are created. Photon energy that has energy less than the energy band-gap of the material will be wasted end with heating the solar cell.

In the design of solar cell, the contact area of electrode should be large and at the same time the exposed area for the incident photon should be very much larger than the area of electrode. The silicon *pn* junction solar cell structure shown in Fig. 7.2 has a *p*-type substrate and *n*-type area exposed to sunlight.
The conversion efficiency of the solar cell can be derived based on the energy band diagram of the solar cell and the idealized equivalent circuit of a solar cell shown in Fig. 7.3.

![Energy band diagram and Idealized equivalent circuit of a solar cell](image)

**Figure 7.2:** Structure of solar cell

**Figure 7.3:** (a) Energy band diagram of a $pn$ junction solar cell and (b) the idealized equivalent of a solar cell

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The current $I_{\text{op}}$ is the photon generated current, $I_s$ is the reverse saturation current of the solar cell, and $I$ is the load current. Thus, from Kirchoff’s current law, $I_{\text{op}}+I = I_s(e^{eV/kT} - 1)$. Thus,

$$I = I_s(e^{eV/kT} - 1) - I_{\text{op}}$$ (7.2)

The rate of electron created is $AL_n g_{\text{op}}$ within electron diffusion length $L_n$ and similar the rate of hole created is $AL_p g_{\text{op}}$ within hole diffusion length $L_p$. $g_{\text{op}}$ is the generation rate of electron-hole pair. Similar carriers are generated within depletion width $W_b$ is $AW_b g_{\text{op}}$. Thus, the resulting optical current due to photon generation is

$$I_{\text{op}} = eA g_{\text{op}} (L_p + L_n + W_b)$$ (7.4)

Since the current is reverse saturation current whereby electron moves to $n$-region, while hole moves to $p$-region, therefore, the overall current of the $pn$ junction shall be

$$I = eA \left( \frac{L_p}{\tau_p} p_{no} + \frac{L_n}{\tau_n} n_{po} \right) (e^{eV/kT} - 1) - eA g_{\text{op}} (L_n + L_p + W_b)$$ (7.4)

It is similar like substitute $I_{\text{op}}$ from equation (7.4) into equation (7.2) and replacing the reverse saturation current as a function of diffusion length, diffusion time, and minority concentrations. Equation (7.4) is also equal to

$$I = I_s \left[ \exp(eV_R/(nkT)) - 1 \right] - I_{\text{op}}$$ (7.5)

where $I_s = eA \left( \frac{L_p}{\tau_p} p_{no} + \frac{L_n}{\tau_n} n_{po} \right)$. This shows a lowering of the normal I-V curve, which depends on the amount of optical generated current $I_{\text{op}}$.

When there is no bias, the short-circuited current $I_{\text{sc}}$ is equal to $I_{\text{op}}$, which is proportional to the EHP-generation rate $g_{\text{op}}$. When open circuit whereby current $I = 0$, the voltage $V_R$ is equal to forward voltage $V_{oc}$, which is

$$V_{oc} = \frac{nkT}{e} \ln \left[ \frac{L_p + L_n + W_b}{(L_p/\tau_p)p_{no} + (L_n/\tau_n)n_{po}} \cdot g_{\text{op}} + 1 \right]$$ (7.6)
Equation (7.6) is also equal to equation (7.7) if we set I = 0 and \( V_R = V_{oc} \) for equation (7.5).

\[
V_{oc} = \frac{n k T}{e} \ln \left[ 1 + \frac{I_{op}}{I_s} \right] \tag{7.7}
\]

The appearance of forward voltage \( V_{oc} \) as the result of illumination is called photovoltaic effect.

For solar cell, the ideality constant \( n \) should be closed to one since there should not have recombination after optical generation of electron-hole pair EHP.

From equation (7.7), the open circuit voltage \( V_{oc} \) is restricted to \( kT/e \), which is less than 1.0V for silicon of band-gap 1.12eV. Thus, individual solar cell is not capable of delivering sufficient power. Thus, solar cells are commonly connected in the form of large arrays and designed having comb-like structure in order to get sufficient exposure to sunlight and deliver enough power.

Figure 7.4 shows the characteristic curve of a solar cell. The power generated is the product of \( I_{max} \) and \( V_{max} \). From equation (7.2), which is \( I = I_s \left( e^{V_m / kT} - 1 \right) - I_{op} \), the power delivered to the load is \( P \), which is

\[
P = IV_R = V_R I_s \left( e^{V_m / kT} - 1 \right) - I_{op} V_R \tag{7.8}
\]

Maximum power is obtained from \( dP/dV_R = 0 \) or from maximum voltage \( V_m \), which is defined as

\[
V_m = \frac{n k T}{e} \ln \left[ \frac{I_{op} / I_S + 1}{e^{V_m / (n k T)} + 1} \right] \cong \frac{V_{oc}}{e} - \frac{kT}{e} \ln \left( 1 + \frac{eV_m}{kT} \right) \tag{7.9}
\]

and maximum current \( I_m \) which is defined as

\[
I_m = I_s \left( \frac{eV_m}{kT} \right) e^{V_m / kT} \cong I_{op} \left( 1 - \frac{1}{e^{V_m / kT}} \right) \tag{7.10}
\]

The maximum power \( P_m \) is then equal to
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\[ P_m = I_m V_m \cong I_{op} \left[ V_{oc} - \frac{kT}{e} \ln \left( 1 + \frac{eV_m}{kT} \right) - \frac{kT}{e} \right] \]  \hspace{1cm} (7.11)

**Figure 7.4:** Characteristic curve of a solar cell

### 7.3 Conversion Efficiency

The conversion efficiency of solar cell can be generally governed by the ideal solar cell design, the sunlight spectrum, and the series resistance and recombination current of the solar cell.

#### 7.3.1 Ideal Efficiency

Many optoelectronic devices have a figure of merit associated with them, which is used to indicate how good the device is. The figure of merit is the fill factor \( FF \) of solar cell is defined as

\[ FF = \frac{P_{max}}{I_{sc} V_{oc}} = \frac{I_{max} V_{max}}{I_{sc} V_{oc}} = 1 - \frac{kT}{eV_{oc}} \ln \left( 1 + \frac{eV_m}{kT} \right) - \frac{kT}{eV_{oc}} \]  \hspace{1cm} (7.12)

where \( I_{sc} \) is the short circuit current, \( V_{oc} \) is the open circuit voltage, \( I_{max} \) and \( V_{max} \) are maximum current and voltage respectively. For most solar cells the fill factor \( \eta \) of 0.6 to 0.8.

The power conversion efficiency for a solar cell is given

\[ P_{eff} = \frac{I_m V_m}{P_m} = \frac{FF \times I_{op} V_{oc}}{P_m} \]  \hspace{1cm} (7.13)
Solar efficiency $\eta_s$ is defined as

$$\eta_s = \frac{P_{\text{max}}}{P_{\text{solar}}} = \frac{FF \times I_{\text{SC}} V_{\text{OC}}}{P_{\text{solar}}}$$  \hfill (7.14)$$

where $P_{\text{solar}}$ is the amount of solar power reaching the cell, which is also equal to the integration of all the photons in the solar spectrum shown in Fig. 7.1. Ideally, $\eta_s$ should be equal to one indicating that all incident solar radiations are being converted to electrical power. In reality it is not the case. The known solar efficiency so far is less than 30% utilizing gallium arsenide.

It is important to note the photon that has energy $h\nu$ smaller than the band-gap $E_G$ will not produce any electron-hole pair EHP. This energy is dissipated as heat. The photon that has energy greater than the band-gap of semiconductor will produce electron-hole pair EHP, excess energy ($h\nu - E_G$) is dissipated as heat. Other factors are reflection and quantum efficiency due to recombination.

In general the solar cell efficiency is low and it depends critically how the semiconductor band-gap matches with the solar energy spectra, which is shown by Plank spectral radiation expression $\rho(\lambda) = \frac{2\pi h c^2}{\lambda^5} \cdot \frac{1}{\exp(hc/\lambda k T) - 1}$ in Fig. 7.1. In Fig. 7.1, it shows that gallium arsenide has a better efficiency than silicon because its cutoff wavelength is closed to the visible high irradiance region of the spectra. However, the technology to produce GaAs solar cell is far more expensive than silicon solar cell. GaAs solar cell is normally used for space application.

### 7.3.2 Spectrum Splitting

The simplest way to improve the power conversion efficiency $P_{\text{eff}}$ is to split the spectrum. By splitting sunlight into narrow wavelength bands and directing each to a cell that has band-gap optimally chosen to convert just this band-gap as shown in Fig. 7.5(a) that using spectrally sensitive mirror, the solar efficiency in principle above 60.0% can be obtained.

Another way to improve the power conversion efficiency $P_{\text{eff}}$ is by stacking the solar cells on top of each other with highest band-gap solar cell at the uppermost side. This arrangement would automatically achieve an identical spectrum splitting effect, making the tandem solar cell approach a reasonably practical way of increasing power conversion efficiency.
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Figure 7.5: Multi-band-gap spectrum concept (a) Spectrum splitting approach and (b) Tandem-cell approach

7.3.3 Series Resistance and Recombination Current

Many other factors degrade the ideal efficiency of solar cell. One of the issue is the series contact resistance $R_s$ from the ohmic loss in the front surface of the cell. The equivalent circuit is shown in Fig. 7.6. From the ideal diode current equation $I = I_s\left(e^{\frac{V_s}{n k T}} - 1\right) - I_{op}$, the I-V characteristic is found to be $I = I_s\left(e^{\frac{V_s - I R_s}{n k T}} - 1\right) - I_{op}$, which is re-written as

$$\ln\left(\frac{I + I_{op}}{I_s} + 1\right) = \frac{e}{kT}\left(V_R - IR_s\right)$$

(7.15)

Figure 7.6: Model of solar cell with series resistance and I-V characteristic curve of non-ideal solar cell
The plots in Fig. 7.6 show two values that are with $R_s = 0\Omega$ and $R_s = 5\Omega$. It can shown that with $5\Omega$ series resistance, the power delivered is about 30% of the maximum power with $R_s = 0\Omega$. The output current and output power are respectively equal to

$$I = I_s \left\{ \exp \left( \frac{e(V_R - IR_s)}{kT} \right) - 1 \right\} - I_{op}$$  \hspace{1cm} (7.16)$$

$$P = I \frac{kT}{e} \ln (\frac{1 + I_{op}}{I_s}) + IR_s$$  \hspace{1cm} (7.17)$$

The series resistance depends on the junction depth, the impurity concentration of the $p$ and $n$-types and the arrangement of the front-surface ohmic contacts. The resistance of $n^+p$ junction is about $0.7\Omega$ and about $0.4\Omega$ for $p^+n$ junction.

Another factor to be considered is the recombination current in the depletion region of the solar cell. For single level center, the recombination current $I_{rec}$ can be expressed as

$$I_{rec} = I_s \left( \exp \left( \frac{eV_R}{2kT} \right) - 1 \right)$$  \hspace{1cm} (7.18)$$

and

$$I_s = A \left( \frac{en_1W_b}{\sqrt{\tau_p\tau_n}} \right)$$  \hspace{1cm} (7.19)$$

$I_s$ is the saturation current. The energy conversion equation can be put into a non ideal form similar to the ideal case by replacing $I_s$ with $I_s'$ and using ideality factor $n = 2$. For silicon solar cell, the recombination current can reduce the efficiency by 25.0%.

### 7.4 Silicon and Compound-Semiconductor Solar Cells

Silicon is the most important semiconductor for solar cells. It is nontoxic and is second most abundant to oxygen in the earth’s crust. Thus, it can be used unlimited. Moreover, the silicon based technology is well established because of its use in microelectronics.
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III-V compound semiconductor and their alloy systems provide wide choices of band-gap with closely matched lattice constants. These compounds are ideal for producing tandem solar cells. For example, AlGa-GaAs, GaInP-GaAs, and GaInAs/InP material system have been developed for solar cells in satellite and space vehicle applications.

7.4.1 Amorphous Silicon Solar Cell

When silicon is deposited by chemical vapor deposition CVD at temperature less than 600°C, an amorphous silicon film is formed regardless of the substrate. Amorphous silicon has large number of energy-state within the band-gap. Owing to short-range crystalline order, the effective mobility is quite small in the order of $10^{-6}$ and $10^{-3}$ cm$^2$/V-s. However, the mobility above $E_c$ and below $E_v$ is between 1.0 to 10.0 cm$^2$/V-s. Owing to the difference of mobility, $E_c$ and $E_v$ are referred to mobility edges and the energy between $E_c$ and $E_v$ is referred to as mobility gap. The mobility gap can be modified by adding impurities. Typical mobility gap of amorphous silicon is 1.7 eV. Amorphous silicon has a very high absorption coefficient, so most sunlight is absorbed with 1.0 µm of surface. Consequently, a very thin layer of amorphous silicon is required for making solar cell. Figure 7.7 shows the density of state for amorphous silicon and Fig. 7.8 shows the energy band diagrams of p-i-n amorphous silicon solar cell.

![Figure 7.7: Density of states of amorphous silicon](image)

Amorphous silicon solar cell has the lowest manufacturing cost and it has modest efficiency of 5.0%.
7.4.2 PERL Cell

The silicon passivated emitter rear locally-diffused PERL solar cell is shown in Fig. 7.9. The cell has inverted pyramids on the top that are formed by using anisotropic etches to expose the slowing etching (111) crystallographic planes. The pyramids reduce reflection of light incident on the top surface since light incident perpendicularly to the cell will strike on one of the inclined (111) planes indirectly and will be refracted obliquely into the cell. The rear contact is separated from the silicon by an intervening oxide layer. This gives much better rear reflection than an aluminum layer. The PERL cell shows high conversion efficiency of 24.0%.

7.4.3 Tandem Solar Cell

Figure 7.10 shows the structure of a tandem solar cell. A $p$-type germanium is used as substrate, which has a lattice constant very close to that of GaAs and
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Ga$_{0.51}$In$_{0.49}$P.A $p$-GaInP layer is then grown to reduce the minority carrier concentration near the rear contact. The bottom junction is the GaAs ($E_G = 1.43$eV) $pn$ junction and the top junction is the GaInP junction ($E_G = 1.9$eV). A tunneling $p^+n^+$ GaAs junction is placed between top and bottom junction to connect the cells. The tandem solar cell has efficiency as high as 30.0% has been achieved.

![Figure 7.10: Monolithic tandem solar cell](image)

7.5 Optical Concentration

Sunlight can be focused by using mirror and lenses. Optical concentration offers an attractive and flexible approach to reducing high cell cost by substituting a concentrator are for much of the cell area. It also offers advantages such as a 20.0% increase in efficiency for a concentration of 1,000 suns, an intensity of $963 \times 10^3$W/m$^2$. Figure 7.11 shows the measured results of a typical silicon solar cell mounted in a concentrated system. Note that the performances improve as the concentration increases one sun toward 1,000 suns. The short-circuit current density increases linearly with concentration. The open-circuit voltage increases at a rate of 0.1V per decade, while the fill factor varies slightly. The efficiency, which is the product of the foregoing three factors divided by the input power, increases at the rate of about 2.0% per decade. With antireflectant coating, the projected efficiency increment of 30.0% at 1,000 suns. Thus, one cell operated under 1,000-sun concentrations can produce the same power output as 1,300 cells under one sun. Potentially optical concentration approach can replace
expensive solar cells with less expensive concentrator materials and related tracking and heat removal system for reducing cost of the system.

Figure 7.11: Efficiency, open-circuit voltage, short-circuit voltage and fill factor versus solar concentration

Exercises

7.1. Calculate the air mass value for a vertical object of height 2.0m and shadow length of 2.5m.

7.2. A Si solar cell has area of 1.0cm² at 300K has $N_A = 5 \times 10^{17}$ cm⁻³, $N_D = 1.2 \times 10^{16}$ cm⁻³, electron diffusion coefficient $D_n = 20$ cm² s⁻¹, hole $D_p = 10$ cm² s⁻¹, electron and hole combination time $\tau_n = 2.8 \times 10^{-7}$ s, $\tau_p = 1.2 \times 10^{-7}$ s and optical generated current $I_{op} = 25$ mA. Calculate the open circuit voltage $V_{oc}$ and optical generation rate $g_{op}$ for the solar cell.

7.3. If the wavelength $\lambda$ of incident photon for Si solar cell mentioned in question 5 is 0.5 µm, calculate the efficiency factor of electron-hole pair EHP creation.
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7.4. If the type of Si solar cell mentioned in question 7.3 is used to power a lighting system in a room that requires 10W at 10V voltage level, how many solar cell of this type is needed? You may assume each solar cell can deliver 90% of its maximum voltage and current, and has a fill factor of 70% respectively.

7.5. A Si solar cell has short-circuit current of 100mA and open-circuit voltage of 0.8V under full illumination. The filled factor is 0.65. What is the maximum power delivered to a load?

7.6. Given the reverse saturation current and optical generated current of a solar cell are 1.0nA and 100mA respectively, calculate the open-circuit voltage and output power at voltage 0.30V of a solar circuit.